Fully Dynamic Broadcasting under SINR

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***Abstract*—Dynamicity is one of the critical characteristics and a major challenge in designing communication protocols in wireless networks. Most of the previous works had focused on the internal node changes (e.g., mobility, arrival, or departure) and not considered the effect of external environmental change. However, the external environmental change, in general, is a more complex phenomenon that can impede nodes from successful communication, implying the protocols of the previous dynamic models do not work well in practice. In this paper, we give an algorithm for distributed broadcasting in a more general model with fully dynamic wireless networks, called FD-Broadcast. Specifically, we present a fully dynamic model which allows node mobility and churns (due to node arrivals/departure) and external environmental change. In contrast to the previous works on dynamic networks, our model defines the full dynamicity in terms of localized topological changes of each node and can tolerate some external environmental change. The external environment changes are captured by the random jamming method. We show that FD-Broadcast can achieve broadcasting in $O(D_S)$ rounds with a high probability guarantee under the assumption of constant dynamic rate in the SINR model, where D_S is the dynamic diameter, a parameter proposed to depict the complexity of dynamic broadcasting. Moreover, the lower bound of dynamic broadcasting is proved to be $\Omega(D_S)$, thus, FD-Broadcast is asymptotically optimal with high probability.**

I. Introduction

Dynamicity is an essential part of wireless networks in real-life and affects the communication capabilities of various mobile devices. Dynamic of wireless network comes from internal node changes (churn and mobility) and external environmental change. Churn, representing nodes arrival and departure, comes from many situations including nodes failure and/or intermittent participation of nodes. Mobility is a basic function in wireless networks, always leading to link changes continually and unpredictably over time in the network topology. External environmental change is quite complicated and might significantly affect the communication among nodes.

Roughly speaking, there are two kinds of external environmental changes, unintentional and intentional. Unintentional environmental changes mainly come from: (a) agents selfishly use the resource without considering other participants, (b) electromagnetic interference from unrelated sources (e.g., transformer, power supply, radar) and (c) others (e.g., movement of objects, temperature increasing). Unintentional environmental changes are not easy to predict, a well adopted idea is to assume a non-trivial adversarial component to represent the dynamicity. Intentional environmental changes, or jamming attacks, can result in more severe problems for communication networks over a shared medium, and are much harder to defend against. Jamming attacks may occur at the physical layer or at the MAC layer. Physical layer jamming may block the communication in some region by transmitting a high power signal. MAC layer jamming may significantly decrease the throughput by destroying the control packet or reserving the channel for maximum allowable number time slots.

We investigating the fundamental communication primitive of the broadcast, which is to disseminate a particular message from a source node to all the other nodes in the network, in fully dynamic wireless networks contain internal nodes' changes and external environmental changes. Designing a MAC protocol under such dynamicity is not trivial and not

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considered in previous works. In fact, even a simple oblivious adversary which jams only a small portion of time slots can block the transmission in the widely used IEEE 802.11 MAC protocol. The SINR (Signal-to-Interference-plus-Noise-rate) model depicts the accumulative and fading features of wireless interference. It defines that the interference fades with distance and the interference is derived from all simultaneously transmitting nodes, not only from nearby nodes. Hence, the SINR model reflects the wireless network in a more precise and accurate way than traditional graph-based models which simplify the interference to be a local and binary phenomenon.

The main contributions of the paper are summarized as follows.

- We propose a fully dynamic network framework under the SINR model that considers localized topological changes of each node and random jamming to capture external environmental changes. The dynamic rate is used to reflect the constrain that describes the magnitude of changing on the local network cumulative transmissions probabilities around each node. In addition to change of nodes, there are some external environmental adversaries that can jam any node individually with probability p $(0 \leq p < 1)$. The fully dynamic model we propose is a more general framework to capture various more realistic dynamic networks, specially, a nature scenario is mobile networks with some adversaries, where the nodes move around unpredictably and nodes are jammed by adversaries.
- We investigate the fundamental task in distributed algorithm, in particular, we focus on the hard case of asynchronous wake up with non-spontaneous broadcasting. For broadcast problem we use a parameter S to depict the time a link can keep stable, and propose a parameter, dynamic diameter D_S, to depict the complexity of dynamic broadcasting. We present a randomized distributed algorithm for fully dynamic broadcasting that need $\Theta(D_S)$ rounds with a high probability guarantee.

The remaining part of this paper is organized as follows. The closely related works are shown in Section II. The network model and definitions are given in Section III. The broadcast algorithm and analysis are presented in Section IV. Finally, the whole paper is concluded in Section V.

II. Related Works

Dynamic networks. Due to the popularity of large-scale wireless devices increased significantly, dynamicity in distributed algorithm has become a very hot topic in wireless networks. During these years, several kinds of dynamic models had been proposed to reflect the dynamicity in wireless network. for churn of nodes [4], [6], [17]. In [17] , Kuhn et al. first proposed the unstructured model to describe the nodes' insertions under unit disk setting. The node crash failures were considered in [6]. Other models mainly focused on the impact of mobility, unreliable links and assumes the statical node set [3], [15], [16], [19]. The dual graph model was introduced in [8]. It defines two graphs on the same node set, one is composed by reliable links, and the other is composed by unreliable links. This model extends the radio networks model to the dynamic case. The T-interval connectivity model given in [16] modeled the dynamic networks in an adversarial manner, under the constraint that the network contains a stable connected spanning subgraph in every interval of T consecutive rounds. Very recently, Yu el al. [25] proposed a dynamic model that admits both node churn and move. However, this model works in a very restricted environment and cannot model various dynamic scenarios. Some surveys on dynamic network models are given in [18]. On the other hand, due to the importance of network security with the advent of Internet-of-Things and the ubiquity of mobile devices, jamming attack in wireless networks has been extensively studied in the field of applied research ([5]). To against (or detect) jamming, the traditional mechanisms are often designed on the physical layer [20]. Some previous works have investigated the protocols on the MAC layer, e.g., coding strategies, channel surfing and spatial retreat. In [23], mechanisms had been designed to encrypt messages and reduce the impact of corrupted messages, or evade the attacker's search. In adversarial attack, two assumptions are considered, one is randomly corrupt messages [21], i.e., each message will be attacked with some probability, the other is randomly jam time slots [1], i.e., each time slot will be jammed with some probability. These theoretical works are more relevant to the studies in this paper.

Broadcasting. Broadcasting is one of the fundamental operations in wireless communication networks and is extensively studied during these years. In static networks, many researches have been focused on broadcasting, in both graph-based model ([10]) and the SINR model ([9]). For non-spontaneous broadcasting under the graph-based model, the best randomized results are $O(D\log(n/D)+\log^2 n)$ [14] and $O(D+\log^6 n)$ [10] without and with collision detection, respectively. Under the SINR model, the best known algorithm was given in [13], with the round cokmplexity $O(D\log^2 n)$.

As for the broadcasting in dynamic networks, under a basic assumption that there exists at least one stable link between nodes with and without the message, Clementi et al. gave a randomized algorithm in [7], which can solve the broadcast problem in $O(n^2/\log n)$ rounds w.h.p., In the dual graph model, an $O(n^{3/2}\sqrt{\log n})$-time deterministic algorithm and an $O(n\log^2 n)$-time randomized algorithm were given in [15].

III. MODELS AND PROBLEM DEFINITION

Consider nodes are arbitrarily placed on two dimensional Euclidean space, possibly in a worst-case fashion. Nodes can transmit messages in rounds, but synchronization is not needed, i.e., global clock is not a must. But we assume that each node is associated with its own clock and the difference of round length among nodes is at most by a factor of 2. Each node is equipped with a half-duplex transceiver. In each round, a node may either transmit a message or sense the channel, but it cannot do both. The nodes communicate via a shared channel. Denote V_t to be the set of nodes at time t

and assume that $|V_t| = poly(n)$ for any time t, where $poly()$ is a polynomial function. Each node is either *active* (state $\mathbb{A}$), or *inactive* (state $\mathbb{I}$). As defined by the non-spontaneous broadcast problem, only active nodes have some massages to be disseminated to other nodes in the network and inactive nodes do nothing except for listening to the channel. For each active node u, its transmission probability at round t is $p_t(u)$, i.e., node u transmits with probability $p_t(u)$, and listens with probability $1 - p_t(u)$.

Notations. Let $d(u, v)$ be the distance between nodes u and v. Node u is called a r-neighbor of v if $d(u, v) \leq r$. Denote by $N_c(v)$ the set of v's cR-neighbors, where R will be determined later. Specifically, if two nodes are R-neighbors, we simply say they are neighbors. The communication graph is a dynamic graph $G_t(V_t, E_t)$, where $(u, v) \in E_t$ if and only if $v \in N_1(u)$ at round t. A set of nodes D is called an r-independent set if for any pair of nodes $u, v \in D$, $d(u, v) > r$. And, if for any node $w \notin D$, there exists a node $u \in D$, such that $d(w, u) \leq r$, then D is called an r-maximal independent set.

Dynamicity. In this paper, we consider a fully dynamic model that not only contain the system internal nodes change (churn and mobility) but also the external environment changes.

For system internal nodes changes, we assume that both churn and mobility of nodes may occur in the network with an unpredictable way. Moved nodes may become new neighbors and changes the network links, which might significantly increase the interference in a short time. Thus, in this work, the mobilities of nodes are assumed to be bounded. We define the dynamic behaviors in the network in a local view that consider the cumulative transmission probabilities in $N_{1/2}(v)$ for any node $v \in V$. W.l.o.g., we may assume that the network changes at the end of every round, and remains unchanged during the round. We define a *dynamic rate* to capture the change of network topology. Consider a period of rounds F and for any round $t \in F$, denote by $P_t(v) = \sum_{u \in N_{1/2}(v)} p_t(u)$ and let $P_t^{\rho}(v)$ be the sum of transmission probabilities of nodes in $N_\rho(v)$ at round t for a constant $\rho > 1$ that will be specified later. Specially if node $u \in \mathbb{I}$, we simple let $p_t(u) = 0$ for convenience. Let $P_t(v)$ and $\hat{P}_t(v)$ denote the sum of the transmission probabilities at the beginning and at the end of round t respectively, then the dynamic rate λ is defined as

$$\lambda = \max_{t \in F, v \in V} \{ | \frac{P_{t+1}(v) - \hat{P}_t(v)}{\hat{P}_t(v)} | \} \tag{1}$$

For external environment changes, as mentioned before, there are various ways to change the environment so that two nodes cannot communicate. In this work, we consider a reasonable random jamming model to capture the environment changes ([2], [12], [21]). In the random jamming model, each listener may be jammed with a constant probability, each sender may be jammed with a constant probability too. The jammed probabilities are independently among nodes and upper bounded by a fixed parameter $p \in [0, 1)$. Whenever nodes are jammed, these nodes will notice a blocked channel and cannot send or receive any useful message. In the mean time, these jammed nodes cannot determine whether the blocked channel is from collision or jamming from the adversary.

With the concept of dynamic rate and random jamming, our dynamic model can fit various dynamic networks.

Stable Diameter. To measure the complexity of dynamic broadcasting algorithm, we introduce the concept of stable path to depict the connectivity of the dynamic networks.

Given a positive integer S as the stable parameter, the S-stable path from node u to node v is defined as follows. For a node sequence $v_0 = u, v_1, ..., v_k = v$, if there is a sequence $I_0, I_1, ..., I_{k-1}$ of time intervals with $I_i = [b_i, e_i]$, such that for each i, $e_i - b_i \geq S$, and $e_i - e_{i-1} \geq S$, nodes v_{i-1} and v_i keep alive and being neighbors during I_{i-1}, then $v_0 \to v_1 \to ... \to v_k$ is a S-stable path. The length of the S-stable path from u to v is then defined as $e_{k-1} - b_0$. Each link on a S-stable path is called a stable link. The stable parameter S depicts the time duration for two nodes to be connected. Larger S means the connection between two nodes can be stable for a longer time. We consider the case that $S \in \Omega(\log n)$, since $\Omega(\log n)$ is the minimum time needed for two nodes to communicate successfully with high probability even if without interference [22]. Note that a stable path may not be connected at any time.

Given the stable parameter S, we define the stable S-distance $D_S(u, v)$ as the minimum length of S-stable paths between u and v. If there is not any S-stable path connecting u and v, then $D_S(u, v) = \infty$. The S-stable diameter of the network is then defined as $D_S = max_{u,v \in V} D_S(u, v)$. If D_S is finite, then the network is called S-stable connected.

Interference Model. Under the SINR model [9], a node can successfully get a message if the strength of the message is strong enough comparing with the noise and the interference. Formally speaking, transmitter u's message can be successfully received by receiver v if and only if

$$\frac{P_u / d(u, v)^\alpha}{N + \sum_{w \in S} P_w / d(w, v)^\alpha} \geq \beta \tag{2}$$

where P_x is the transmission power of node x, $\alpha > 2$ is the pass loss exponent, N is the ambient noise, $\beta \geq 1$ denotes the minimum signal to interference and noise ratio required for decoding a message, S is node set in $V \setminus \{u, v\}$ that are currently transmitting.

In this paper, we assume the *uniform power assignment*, i.e., all nodes take the same transmission power P. Uniform power assignment is one of the most common power assignments in practice. With the help of physical carrier sensing, nodes can detect the interference when listening to the channel and determine the distance from the sender if successfully receives a message.

Denote $R_T = (P/\beta N)^{1/\alpha}$ to be the *maximal transmission range*, which is the maximum possible distance a node can successfully receive a message from another node. However, even with a little interference, two nodes with distance R_T cannot communicate. Thus, the links that are used for communication have to be 'stronger' than those defined by the transmission range. We define a *communication range* R with

$R = (1-\epsilon)R_T$, where $\epsilon \in (0,1)$ is a constant determined by the environment.

For a successful message transmission within a distance R, the following inequality must be hold.

$$N + \sum_{w \in S \setminus \{u\}} P_w/d(w,v)^\alpha \le P/\beta R^\alpha.$$

Assume that the ambient noise level N is upper bounded by a fraction of the maximum tolerable interference level. Let $N \le \frac{P}{2\beta R^\alpha} \cdot (\frac{2}{(\rho+1)^\alpha} - \frac{1}{2} \cdot (\frac{\rho}{\rho+1})^{\frac{\alpha-2}{2}} \cdot k)$, where ρ is a large constant determined in the analysis and $k < \min\{2, \frac{4}{\rho^{(\alpha-2)/2} \cdot (\rho+1)^{(\alpha+2)/2}}\}$.

Problem. In the fully dynamic multi-hop wireless ad hoc network, a source node has a message M_s to be sent to all the other nodes, the objective is to minimize the broadcasting time.

In the following, we give some useful facts which will be used in the latter analysis.

Fact 1: Given a set of probabilities $p_1, ..., p_n$, where $p_i \in [0, \frac{1}{2}]$ for all i , the following inequalities hold:

$$(1/4)^{\sum_{k=1}^n p_k} \le \prod_{k=1}^n (1-p_k) \le (1/e)^{\sum_{k=1}^n p_k}.$$

Fact 2: Consider two disks D_{R_1} and D_{R_2} where $R_1 > R_2$. Define χ^{R_1,R_2} to be the smallest number of disks D_{R_2} needed to cover a large disk D_{R_1}. Because the ratio between the area of large disk D_{R_1} and the area of small disks D_{R_2} is $2\pi/3\sqrt{3}$ [11], and all small disks D_{R_2} intersecting D_{R_1} are completely inside the area of radius $R' = R_1 + 2R_2$, it holds that

$$\chi^{R_1,R_2} \le \frac{2\pi}{3\sqrt{3}} \cdot \frac{(R_1+2R_2)^2}{R_2^2}.$$

Fact 3: Given a set of random variables $X_1, ..., X_n$. Suppose that there are values $p_1, ..., p_n$, ($p_i \in [0,1]$ for all i), with $E[\prod_{i \in S} X_i] \le \prod_{i \in S} p_i$ for every set $S \subseteq \{1, ..., n\}$. Then it holds for $X = \sum_{i=1}^n X_i$, $\mu = \sum_{i=1}^n p_i$ and any $\delta > 0$ that

$$P[X \ge (1+\delta)\mu] \le [\frac{e^\delta}{(1+\delta)^{1+\delta}}]^\mu \le e^{-\frac{\delta^2 \mu}{2(1+\delta/3)}}.$$

If, on the other hand, it holds that $E[\prod_{i \in S} X_i] \ge \prod_{i \in S} p_i$ for every set $S \subseteq \{1, ..., n\}$. Then it holds for any $0 < \delta < 1$ that

$$P[X \le (1-\delta)\mu] \le [\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}]^\mu \le e^{-\frac{\delta^2 \mu}{2}}.$$

IV. BROADCAST PROTOCOL

In this section, we present the algorithm to handle the broadcast problem in fully dynamic wireless networks. We call the algorithm FD-Broadcast, which stand for fully dynamic broadcasting. Moreover, the algorithm is proved asymptotical optimal by showing the matched upper and lower bounds of the accomplish time.

A. Algorithm

The broadcasting algorithm is to disseminate the message by letting an active node locally broadcast the source message to its neighboring inactive node, until the message is successfully received by all nodes in the network. If more than one transmission happens in a round and the signal strengths at a node v are large enough, node v cannot successfully receive the message since the SINR values from each transmitter is less than β.

If the contention in some area is high, the active nodes' cumulative transmission probabilities in this local region is high due to dynamicity or initially setting, the message dissemination in this region might be hindered due to plenty of collisions. On the other hand, if the contention in some area is low, according to Fact 1, the probability of no transmission is large and the message cannot be successfully transmitted too. However, if the active nodes' cumulative transmission probabilities in some area is a constant, not too large and not too low, by some simple computation, the probability of successfully transmission is a constant if the receiver and the transmitter are not jammed. Hence, the critical core of handling the broadcast problem is balancing the contention in each local region.

In our algorithm, a randomized contention balancing procedure is implemented. Each node adjusts its transmission probability according to the environmental status. Formally speaking, every node $v \in \mathbb{A}$ determines to transmit with a probability $p_t(v) \le p_{max}$ in every round, which is initialized as $p_t(v) = p_{max}$ when v start the algorithm. $p_t(v)$ will be doubled after the slot if v senses an idle channel and be halved in other cases. A node detects a *busy* channel if the interference it sensed exceeds the threshold $T_b = P \cdot R^{-\alpha}$ or jammed.

For the convenience of analysis, we set the parameters in the algorithm as follows: $p_{max} = 1/2$ (actually, p_{max} can be set to any constant satisfying $0 < p_{max} \le 1/2$); $\hat{p} = \log_4 \frac{10(1-p)}{9}$; $p_{min} = \frac{1}{4n}$; $\gamma = 20 \cdot \max\{\frac{4^g}{(1-p)^2 p_{max}}, \frac{4^{\chi^{(\rho+1)R,\rho R} \cdot g}}{(1-p)^2 \hat{p}}\}$, where g is sufficiently large constant determined in the analysis. $\lambda \le 2^{1-w} - 1$; $w = \frac{16(1+\delta)(\rho+1/2)^2}{\sigma} e^{-\phi}$; ϕ, δ and σ are constants and will be determined later.

The time of algorithm implementation is bounded and asymptotically optimal when the stable parameter $S \ge \gamma \log n$.

By the randomized contention balancing from Algorithm FD-Broadcast, we will show in Lemma 7 that in most rounds of an interval $I = \Omega(\gamma \log n)$, there exists sufficiently large constants ρ and g such that for each node v, the following two properties hold.

1) *Bounded Contention.* $P_t^\rho(v) \le g$ for specified constants ρ and g, and
2) *Bounded Interference.* The expected interference at v is upper bounded by T for a specified constant T.

Formally, let $\bar{I}_t^\rho(v)$ be the expected interference at node v that are caused by nodes outside $N_t^\rho(v)$, i.e., $\bar{I}_t^\rho(v) = \sum_{u \notin N_t^\rho(v)} S_{u,v} \cdot p_t(u)$, where $S_{u,v} = P/d(u,v)^\alpha$. Thus in most rounds of an interval $I = \Omega(\gamma \log n)$,

Algorithm 1: FD-Broadcast: for each active node v

Initially, $p_0(v) = p_{max}$;
In round t, v does:
Let $X \leftarrow 1$ or 0 with probability $p_t(v)$ and $1 - p_t(v)$, respectively.
if $X = 1$ **then**
    Transmit the message; $p_{t+1}(v) \leftarrow \max\{\frac{p_t(v)}{2}, p_{min}\}$;
else
    Listen to the channel:
    if *detected* busy *channel* **then**
        $p_{t+1}(v) \leftarrow \max\{\frac{p_t(v)}{2}, p_{min}\}$;
    else
        $p_{t+1}(v) \leftarrow \min\{2p_t(v), p_{max}\}$

1) $P_t^{\rho}(v)$ is upper bounded by a constant g and
2) $\bar{I}_t^{\rho}(v)$ is upper bounded by T, where $T = \frac{P}{4\beta R^{\alpha}} \leq \frac{T_b}{10}$ when $\beta \geq 2.5$.

In $O(\gamma \log n)$ rounds, if the surrounding contention of a node v is small for sufficiently large number of rounds, the transmission probability of node v will be doubled many times and may achieves p_{max} a constant portion of $O(\gamma \log n)$ rounds. Thus, node v might successfully transmit the message to its neighbors in $O(\gamma \log n)$ rounds. If the surrounding contention of a node v is a constant, not too small, for sufficiently large number of rounds, some active neighbor of v might transmit the message to inactive neighbor of v. Under the fully dynamic environment, each node v will be jammed independently with the probability p. The above statement will be proved in Lemma 8.

Setting the stable parameter $S \geq \gamma \log n$, we will show (Theorem 1) that the message M_s can be successfully transmitted from the source node u to any other node v if there exist an S-stable path from u to v. Moreover, Algorithm FD-Broadcast is asymptotic optimal (Theorem 2) by showing $\Omega(D_S)$ rounds is needed for successful transmission w.h.p. in a constructed structure.

B. Analysis

In this part, we analyze the round complexity and correctness of Algorithm FD-Broadcast. Let ρ be a constant defined as follows.

$$\rho = [\beta \cdot 2^{5+\alpha'/2}\pi\phi \cdot \frac{1}{\alpha'}]^{\frac{2}{\alpha'}}, \text{where } \alpha' = \alpha - 2 \tag{3}$$

If $P_t(v) \geq \phi$, $P_{t+1}(v)$ will be decreased with large probability in the next round.

Lemma 1: In a round t and for a node v, if $P_t(v) \geq \phi$, then $\Pr[P_{t+1}(v) \leq \frac{(1+\lambda)P_t(v)}{2}] \geq 1 - e^{-\phi}$.

Proof: We need to analyze the probability that each node in $N_{1/2}(v)$ halves its transmission probability. By the detection threshold T_b, if there is at least one node in $N_{1/2}(v)$ transmitting or all nodes in $N_{1/2}(v)$ jammed, then all nodes in $N_{1/2}(v)$ will sense a busy channel. Denote by $\mathcal{E}_1$ and $\mathcal{E}_0$ the events that there is at least one node in $N_{1/2}(v)$ transmitting and there are no nodes in $N_{1/2}(v)$ transmitting, respectively. We have $\Pr[\hat{P}_t = P_t/2] \geq \Pr[\mathcal{E}_1] = 1 - \Pr[\mathcal{E}_0]$.

We next bound $\Pr[\mathcal{E}_0]$. The probability that a node $u \in N_{1/2}(v)$ does not transmit is $1 - p_t(u)$. Thus, the probability that no node transmits in round t is equal to

$$\begin{aligned}\Pr[\mathcal{E}_0] = &\prod_{v \in N_{1/2}(v)} (1 - p_t(v)) \leq \prod_{v \in N_{1/2}(v)} e^{-p_t(v)} \\ &= e^{-P_t(v)} \leq e^{-\phi}.\end{aligned} \tag{4}$$

So $\Pr[\hat{P}_t(v) = P_t(v)/2] \geq 1 - \Pr[\mathcal{E}_0] \geq 1 - e^{-\phi}$.

Due to the dynamicity, we have

$$(1 - \lambda)\hat{P}_t(v) \leq P_{t+1}(v) \leq (1 + \lambda)\hat{P}_t(v).$$

Thus,

$$\begin{aligned} & \Pr[P_{t+1}(v) \leq \frac{(1+\lambda)P_t(v)}{2}] \\ \geq \; & \Pr[\frac{(1-\lambda)P_t(v)}{2} \leq P_{t+1}(v) \leq \frac{(1+\lambda)P_t(v)}{2}] \\ \geq \; & 1 - e^{-\phi} \end{aligned}$$

■

We next consider the variant of $P_r(v)$ in an interval I of size $\Theta(\gamma \log n)$ for a particular node v. W.l.o.g., let $I = \{1, \ldots, t\}$. Assume initially $P_0(v) = \phi_0$. It is clearly that $\phi_0 \leq n$. We can show that during a sufficiently large interval there is a time satisfy $P_r(v) \leq \phi$ w.h.p..

Lemma 2: For any node v and any constant $\phi \geq 2$, executing Algorithm FD-Broadcast for $\Theta(\gamma \log n)$ rounds, there is a round $r \in I$ satisfying $P_r(v) \leq \phi$ w.h.p.

Proof: In each rounds, the transmission probability of each node either double or half. $P_r(v)$ will be either increasing or decreasing for each round $r \in \{1, ..., t\}$. Since $P_0(v) = \phi_0 \leq n$ and $\phi \geq 2$, if the number of decreasing rounds is sufficiently larger than the number of increasing rounds, we can claim that there is a round $r \in I$ satisfying $P_r(v) \leq \phi$.

For round $r \in I$, define the binary random variable Y_r as follows.

$$Y_r = \begin{cases} 0 & \text{if } P_r(v) < \phi \text{ or} \\ & P_r(v) \geq \phi \text{ and } P_{r+1}(v) \leq \frac{(\lambda+1)P_r(v)}{2} \\ 1 & \text{otherwise} \end{cases}$$

Thus,

$$\begin{aligned} \Pr[Y_r = 0] &= \Pr[P_r(v) < \phi] + \\ & \quad \Pr[P_r(v) \geq \phi \wedge P_{r+1}(v) \leq \frac{(\lambda+1)P_r(v)}{2}] \\ &= \Pr[P_r(v) < \phi] + \Pr[P_r(v) \geq \phi] \cdot \\ & \quad \Pr[P_{r+1}(v) \leq \frac{(\lambda+1)P_r(v)}{2} | P_r(v) \geq \phi] \\ &\geq \Pr[P_r(v) < \phi] + \Pr[p_r \geq \phi] \cdot (1 - e^{-\phi}) \\ &\geq 1 - e^{-\phi}, \end{aligned}$$

From the above analysis, we have $\Pr[Y_r = 1] \leq e^{-\phi}$ for all $r \in I$. Furthermore, for any set $S \subseteq I$.

$$E[\prod_{r \in S} Y_r] \leq (e^{-\phi})^{|S|}. \tag{5}$$

Let $t = \Theta(\gamma \log n)$ and $Y = \sum_{r=1}^{t} Y_r$, $P_{r+1}(v) \leq 2(\lambda + 1)P_r(v)$ is obvious for any case even the dynamic of nodes. Assume that there is no $P_r(v) < \phi$ for all $r \in I$. Thus,

$$\phi_0 \cdot (2(\lambda+1))^Y \cdot (\frac{\lambda+1}{2})^{t-Y} \geq \phi,$$

which implies the number of rounds satisfies $Y_r = 1$ is at least $((1 - \log(\lambda+1))t - \log\lceil\phi_0/\phi\rceil)/2 = \Omega(\gamma \log n)$. This equation holds since $\phi_0 \leq n$ and $\phi \geq 2$. Let S be such rounds with $Y_r = 1$, according to Equation (5),

$$E[\prod_{r \in S} Y_r] \leq (e^{-\phi})^{|S|} = O(n^{-\phi}).$$

Let $\mu = |I| \cdot e^{-\phi}$. From the Chernoff Bounds,

$$\Pr[Y > (1+\delta)\mu] \leq e^{-\frac{\delta^2 \mu}{2(1+\delta/3)}}$$

for any constant δ. By choosing sufficiently large constant γ, the number of rounds with $Y_r = 1$ can be small enough, i.e., $S \leq |I|/3$ w.h.p. In another words, $|I| > 2|S| + \log n$ w.h.p. Therefore, there are sufficiently large number of rounds to decrease $P_r(v)$ and make $P_r(v) \leq \phi$ for some $r \in I$ w.h.p. ■

Let $Y = \sum_{r=1}^{t} Y_r$ where Y_r is the variable defined as above. From Lemma 1 and Lemma 2, $P_r(v)$ is limited by the contention balancing strategy. Next we show that in sufficiently large interval I, the expected number of rounds r satisfying $P_r(v) \geq \phi$ can be as less as we want.

Under the fully dynamic environment, $P_{t+1}(v) < 2P_t(v)(\lambda+1)$. Combining with Lemma 2, we have

Lemma 3: The number of rounds $r \in I$ with $P_r(v) \geq \phi$ is upper bounded by $\frac{2Y + \log\lceil\phi_0/\phi\rceil + 2}{1 - \log(\lambda+1)}$.

Due to page limit, the detailed proof of Lemma 3, 4, 5, 6 and 7 can be found in [24].

Actually, in interval I, the expected number of rounds r satisfying $P_r(v) \geq \phi$ can be as less as we want. From the Chernoff bound, we can prove the following conclusion.

Lemma 4: For any time interval I with $\sum_{u \in D_{1/2}(v)} p_0(u) = \phi_0$, let X_ϕ be the random variable on the number of rounds $r \in I$ satisfying $P_r(v) \geq \phi$. For any $\phi \geq 2$ and $\delta \geq 2$, the following inequality holds

$$\Pr[X_\phi \geq \frac{(1+\delta)2|I|/e^{\phi} + \log\lceil\phi_0/\phi\rceil + 2}{1 - \log(\lambda+1)}]$$
$$\leq (e/(1+\delta))^{\delta|I|/e^{\phi}}$$

The bound shown in Lemma 4 is not obvious since there is a term $\log \phi_0$, which can be as large as $\log(np_{max})$. According to Lemma 2, we know that for any time interval with length $\Theta(\gamma \log n)$, there exist a round r in this interval satisfying $P_r(v) \leq \phi$ w.h.p. for any $\phi \geq 2$. Consider two intervals, $I_1 = [0, r_0]$, $I_2 = [r_0 + 1, 2r_0]$. In interval I_1, there is a round r such that $r_0 - r = O(\log n)$ and $P_r(v) \leq \phi$ w.h.p. Let $I = [r, r_0] \bigcup I_2$, the length of I is $\Theta(\gamma \log n)$, satisfying the condition in Lemma 2. Thus, the additive term of $\log \phi_0$ can be avoided.

Lemma 5: Consider a time interval I starting at round t_0, suppose both $|I|$ and t_0 are sufficiently large, say $\Omega(\gamma \log n)$. Let X_ϕ be a random variable on the number of rounds in I satisfying $P_r(v) \geq \phi$. For any $\phi \geq 2$, it holds that

$$E[X_\phi] \leq \frac{1}{1 - \log(\lambda+1)}(40|I|e^{-\phi} + 2).$$

Based on the above transmission probability analysis, we will show that the number of rounds with large interference can be sufficiently small. For any time interval I with $|I| = \Omega(\gamma \log n)$, let Z be the number of intervals with interference no less than T, i.e., $\bar{I}_t^{\rho} \geq T$. Partition the area outside $D_\rho(v)$ into rings such that the difference between the outer radius and the inner radius is exactly 1. Each ring will be further partitioned into sectors such that the distance between any two nodes in the same sector is at most R. Sum up the interference on all sectors, we have the following lemma.

Lemma 6: $\Pr[Z \geq \sigma|I|]$ can be polynomially small in n for any constant $\sigma > 0$.

For a node v, a round t in I is called *good* if and only if $P_t^{\rho}(v) \leq g$ for some fixed constant g and $\bar{I}_t^{\rho}(v) \leq T$.

Now, we can get the result from the contention balancing strategy that for each node, most rounds are good.

Lemma 7: In the fully dynamic environment, for any constant $\sigma > 0$, at least $(1-\sigma)|I|$ of the rounds in I are good for v, w.h.p., if g and ρ are sufficiently large.

For the convenience of analysis, we divide the time into phases of $\gamma \log n$ rounds, where γ is a sufficiently large constant. The phases are further classified into two types: *high* and *low*. A phase is called *high* if in at least $1/10$-fraction of the rounds, $P_t^{\rho}(v) \geq \hat{p}$ for some small constant $\hat{p}$. Otherwise, the phase is called *low*.

Lemma 8: If the dynamic rate $\lambda \leq 2^{1-w} - 1$ and the stable parameter $S \geq \gamma \log n$, then a node $u \in \mathbb{I}$ can receive the message M_s at the end of I, w.h.p., if its has a stable link with an active node v.

Proof: From the previous lemmas, $1 - \log(\lambda+1)$ should be a constant. When the dynamic rate $\lambda \leq 2^{1-w} - 1$, these lemmas are justified.

Case 1: If the phase is low phase for v.

Let $\sigma = \frac{1}{20}$. Then by Lemma 7, there are at least $(1-\sigma)$-fraction of rounds in I that are good for v. By the definition of low phase, we can then obtain that in $\frac{4}{5}$-fraction of rounds in I that both good and low contention for v. Denote the set of these rounds as I'. We next consider the rounds in I'.

For a round $t \in I'$, denote by $\mathcal{E}_1$ the event that the interference at v from nodes outside $N_\rho(v)$ is at most T_b, and by $\mathcal{E}_2$ the event that there is not any transmitter within distance ρR from v and v is not jammed. When $\mathcal{E}_1$ and $\mathcal{E}_2$ occur, v will sense an idle channel, and make its transmission probability double. We next bound the probability that these two events occur.

Because t is a good round, the expect interference at v from nodes outside $N_\rho(v)$ is at most $T \le T_b/10$. Using Markov Inequality,

$$Pr(\mathcal{E}_1) \ge 9/10. \tag{6}$$

In t, we know that $P_t^\rho(v) \le \hat{p}$. We have

$$\begin{aligned} Pr(\mathcal{E}_2) &= (1-p)\prod_{u\in N_\rho(v)}(1-p_t(u)) \\ &\ge (1-p)(\frac{1}{4})^{\sum_{u\in N_\rho(v)} p_t(u)} \\ &\ge (1-p)(\frac{1}{4})^{\hat{p}}. \end{aligned}$$

Hence, $Pr(\mathcal{E}_2) \ge 9/10$.

Combining the above results, the probability that v senses an idle channel is at least $Pr(\mathcal{E}_1)\cdot Pr(\mathcal{E}_2) \ge 4/5$. Then, in expectation, there are at least $\frac{4}{5}$-fraction of rounds in I' in which v senses an idle channel. Using Chernoff bound, it can be shown that w.h.p., v senses an idle channel in at least $\frac{7}{10}$-fraction of rounds in I'. This means that v doubles its transmission probability in at least $\frac{7}{10}\cdot\frac{4}{5} = \frac{28}{50}$ fraction of rounds in I. Notice that at the beginning of I, $p_t(v) \ge p_{min} = \frac{1}{4n}$. And for other rounds, $p_t(v)$ is halved in the worst case. Then if γ is sufficiently large, there will be $\frac{28}{50} - \frac{22}{50} - \frac{1}{50} = \frac{1}{10}$ fraction of rounds in I in which v attains the maximum transmission probability p_{max}, where the $\frac{1}{50}|I|$ doubling is used for increasing the initial transmission probability to the maximum one. As above, we have shown that in a low phase, v will attain the maximum transmission probability in $\frac{1}{10}$-fraction of rounds in I w.h.p. Thus, there are at least $\frac{1}{20}$-fraction of rounds in I that both good for node u and $p_v = p_{max}$. Denote the set of these rounds as I'', we next show that at any $t \in I''$, with constant probability, the message sent by node v will be received successfully by node u.

Consider a round $t \in I''$, since t is a good round for node u, the expected value of interference at the intended receiver u, caused by transmissions outside $N_\rho(u)$ is at most $\frac{P}{4\beta R^\alpha}$. We can use Markov inequality to show that the probability that the interference at u caused by transmissions outside $N_\rho(u)$ exceeds $k\cdot\frac{P}{4\beta R^\alpha}$ is less than 1/k. Consequently, if v is the only transmitter in $N_\rho(u)$, with probability $P_{SINR\ge\beta} \ge 1/k$, the SINR at the intended receiver u can be lower bounded by

$$SINR_{v,u} = \frac{\frac{P}{d(v,u)^\alpha}}{k\cdot\frac{P}{4\beta R^\alpha} + N} \ge \beta.$$

The above inequality holds since $d(v,u) \le R$ and ambient noise N is upper bounded by $\frac{P}{2\beta R^\alpha}\cdot(\frac{2}{(\rho+1)^\alpha} - \frac{1}{2}\cdot(\frac{\rho}{\rho+1})^{\frac{\alpha-2}{2}}\cdot k)$. The probability that v is the only transmitter in $N_\rho(u)$ is at least

$$p_v \prod_{w\in N_\rho(u)\setminus\{v\}}(1-p_w) \ge p_{max}(\frac{1}{4})^g$$

Combining the above analysis, the probability that node u successfully receives message from node v at a time slot is

$$\begin{aligned} P_{success} &\ge (1-p)^2\cdot P_{SINR\ge\beta}\cdot p_{max}(\frac{1}{4})^g \\ &\ge \frac{(1-p)^2}{k}\cdot p_{max}(\frac{1}{4})^g \end{aligned}$$

Thus, the probability P_{fail} after $\frac{\gamma\log n}{20}$ rounds is

$$P_{fail} \le (1 - \frac{(1-p)^2}{k}\cdot p_{max}(\frac{1}{4})^g)^{\frac{\gamma\log n}{20}} \le 1/n^k$$

where $\gamma = 20\cdot\max\{\frac{4^g}{(1-p)^2 p_{max}}, \frac{4^{\chi^{(\rho+1)R,\rho R}\cdot g}}{(1-p)^2\hat{p}}\}$.

Case 2: If the phase is high phase for v.

By Fact 2, Lemma 4, Lemma 6 and Lemma 7, there are at least $(1-\sigma')$-fraction of rounds in I that $P_t^{\rho+1}(u) \le g\cdot\chi^{(\rho+1)R,\rho R}$ and the expected value of interference at the intended receiver u, caused by transmissions outside $N_{(\rho+1)}(u)$ is at most $T\cdot(\frac{\rho}{\rho+1})^{(\frac{\alpha-2}{2})}$. Let $\sigma' = \frac{1}{20}$. Since this is a high phase for node v, we can then obtain that in $\frac{1}{20}$-fraction of rounds in I that $\hat{p} \le P_t^{(\rho+1)}(u) \le g\cdot\chi^{(\rho+1)R,\rho R}$ and $\bar{I}_t^{\rho+1}(u) \le T\cdot(\frac{\rho}{\rho+1})^{(\frac{\alpha-2}{2})}$. Denote the set of these rounds as I'''.

Consider a round t in I'''. Similar to Case 1, we can use Markov inequality to show the probability that the interference at u caused by transmissions outside $N_{(\rho+1)}(u)$ exceeds $k\cdot T\cdot(\frac{\rho}{\rho+1})^{(\frac{\alpha-2}{2})}$ is less than $1/k$. Consequently, provided that v' is the only node transmission in $N_{(\rho+1)}(u)$, with probability $P_{SINR\ge\beta} \ge 1/k$, the SINR at the intended receiver u can be lower bounded by

$$SINR_{v',u} = \frac{\frac{P}{d(v',u)^\alpha}}{k\cdot T\cdot(\frac{\rho}{\rho+1})^{(\frac{\alpha-2}{2})} + N} \ge \beta.$$

The above inequality holds since $d(v',u) \le (\rho+1)R$ and ambient noise N is upper bounded by $\frac{P}{2\beta R^\alpha}\cdot(\frac{2}{(\rho+1)^\alpha} - \frac{1}{2}\cdot(\frac{\rho}{\rho+1})^{\frac{\alpha-2}{2}}\cdot k)$, if v' is the only transmitter in $N_{(\rho+1)}(u)$. The probability that v' is the only transmitter in $N_{(\rho+1)}(u)$ is at least

$$\begin{aligned} &\sum_{v'\in N_{(\rho+1)}(u)} p_{v'} \prod_{h\in N_{(\rho+1)}(u)\setminus\{v'\}}(1-p_h) \\ &\ge \hat{p}(\frac{1}{4})^{g\cdot\chi^{(\rho+1)R,\rho R}} \end{aligned}$$

Combining the above analysis, the probability that node u successfully receives message from a node $v' \in N_{(\rho+1)}(u)$ at a time slot is

$$\begin{aligned} P_{success} &\ge (1-p)^2\cdot P_{SINR\ge\beta}\cdot\hat{p}(\frac{1}{4})^{g\cdot\chi^{(\rho+1)R,\rho R}} \\ &\ge \frac{(1-p)^2}{k}\cdot\hat{p}(\frac{1}{4})^{g\cdot\chi^{(\rho+1)R,\rho R}} \end{aligned}$$

The probability P_{fail} after $\frac{\gamma\log n}{20}$ rounds is

$$P_{fail} \le (1 - \frac{(1-p)^2}{k}\cdot\hat{p}(\frac{1}{4})^{g\cdot\chi^{(\rho+1)R,\rho R}})^{\frac{\gamma\log n}{20}} \le 1/n^k$$

where $\gamma = 20 \cdot \max\{\frac{4^g}{(1-p)^2 p_{max}}, \frac{4^{\chi(\rho+1)R,\rho R}\cdot g}{(1-p)^2 \hat{p}}\}$.

Therefore, if a node $u \in \mathbb{I}$ has a stable link with an active node v, it can receive node v's message at the end of I, w.h.p. ■

Theorem 1: If the dynamic rate $\lambda \leq 2^{1-w} - 1$ and the stable parameter $S \geq \gamma \log n$, each node can get the message M_s of the source node in $O(D_S)$ rounds w.h.p.

Proof: From lemma 8, if an inactive node u has a stable link with an active node v, it can receive the message M_s, w.h.p. within $O(\gamma \log n)$ rounds. We then consider how long it takes from the beginning of the algorithm till a node v receives the message. Let $L = \{L_1, L_2, ..., L_k\}$ be the stable path with length $D_S(s, v)$ between the source s and v. By the definition of the stable path, the stable links in L keep stable for S rounds successively. Then based on the above analysis, for each stable link $L_k = (u_k, v_k)$ with $1 \leq k \leq d$, v_k will get the message M_s after u_k becomes active for $O(\gamma \log n)$ rounds, w.h.p. Hence, after at most $O(D_S(s, v))$ rounds, node v will receive M_s w.h.p. Therefore, after $O(D_S)$ rounds, all nodes can receive the message M_s of the source node, w.h.p. ■

C. Lower Bound

Theorem 2: There exists a dynamic graph satisfying the above stable parameter such that any algorithm needs $\Omega(D_S)$ rounds to complete the broadcast w.h.p.

Proof: Consider $2n$ node sets V_0, V_1, ..., V_{2n}, where V_{2k} contains n nodes while $V_{2k+1} = \{v_{2k+1}\}$. Nodes in V_{2k} have the same fixed position p_{2k}. The distance between p_{2k} and p_{2k+2} is $3R$. Node v_{2k+1} has two possible positions, p_{2k+1} and p'_{2k+1} such that $d(p_{2k}, p_{2k+1}) = d(p'_{2k+1}, p_{2k+2}) = R$ and $d(p_{2k}, p'_{2k+1}) = d(p_{2k+1}, p_{2k+2}) = 2R$. In rounds $[2\ell\gamma \log n, (2\ell+1)\gamma \log n - 1)$, v_{2k+1} is in p_{2k+1}, while in rounds $[2\ell+1)\gamma \log n, (2\ell+2)\gamma \log n - 1)$, v_{2k+1} is in p'_{2k+1}, where ℓ is any non-negative integer. From these descriptions, a linear dynamic structure is constructed. Suppose the source node s is with distance R to V_0, thus, in a single round, the message will be sent to all nodes in V_0. However, it takes $\Omega(\log n)$ rounds for v_1 to successfully receive this message w.h.p. Note that v_1 stays in p_1 during $[0, \gamma \log n)$ rounds. This time can guarantee v_1 gets the message w.h.p. But nodes in V_2 cannot receive the message during this time.

The following procedures are similar. From the definition D_S of the length of S-stable path, the time to guarantee all nodes in this linear dynamic network get the message w.h.p is at least $\Omega(D_S)$. ■

V. Conclusion

In this paper, an algorithm is given to handle the broadcast problem in fully dynamic wireless networks. We consider local change of nodes and external environmental changes, which suit the fashion of distributed algorithms. With the help of physical carrier sensing, the algorithm is proved to be asymptotically optimal. The fully dynamic model is potentially useful for solving many other fundamental problems in wireless networks, such as local broadcast and data aggregation, which are the interesting future directions in this area.

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