2012 8th IEEE International Conference on Distributed Computing in Sensor Systems

An $O(\log n)$ Distributed Approximation Algorithm for Local Broadcasting in Unstructured Wireless Networks

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***Abstract*—The unstructured multi-hop radio network model, with asynchronous wake-up, no collision detection and little knowledge on the network topology, is proposed for capturing the particularly harsh characteristics of initially deployed wireless ad hoc and sensor networks. In this paper, assuming such a practical model, we study a fundamental problem of both theoretical and practical interests—the local broadcasting problem. Given a set of nodes V where each node wants to broadcast a message to all its neighbors that are within a certain local broadcasting range R, the problem is to schedule all these requests in the fewest timeslots. By adopting the physical interference model and without any knowledge on neighborhood, we give a new randomized distributed approximation algorithm for the local broadcasting problem with approximation ratio $O(\log n)$ where n is the number of nodes. This distributed approximation algorithm improves the state-of-the-art result in [22] by a logarithmic factor.**

I. Introduction

Newly formed wireless ad hoc and sensor networks lack a structure that is known a priori. In order to construct such a structure or perform any tasks on such networks, each node must coordinate with their neighbors by communicating with each other, which gives rise to the local broadcasting problem, i.e., each node needs to broadcast a message to its neighbors within some pre-defined local broadcasting range. In single-hop networks, it is easy to see that the local broadcasting problem is the same as the traditional gossiping problem.

Algorithmic study on the local broadcasting problem was first motivated by simulating the traditional synchronous message passing model in [1]. The message passing model however abstracts away many crucial elements in radio networks, such as interference, collision and asynchrony. Such a model is mainly used to ease the understanding of the essential aspects of the problems at hand, and many novel distributed algorithms are presented assuming this model. When further considering interference and collision, an obvious approach is to simulate each single round of the original algorithm in the message passing model. More precisely, by performing a local broadcast in each simulating phase, it is ensured that every message passed in the original algorithm during the simulated round can be successfully transmitted. Although there have been many distributed algorithms that are based on the message passing model, there are relatively few efficient ones that work for many of the fundamental problems in radio networks, let alone using the more practical physical interference model.

Despite the graph based interference model was first adopted for studying the local broadcasting problem, it cannot fully depict the realistic interferences in radio networks. The graph based model defines the interference as a localized function. A transmission can only be interfered by nearby simultaneous transmissions. In reality, however, the interference is cumulative by considering all simultaneous transmissions, not only the nearby ones. Such a reality is captured by the physical interference model—the Signal-to-Interference-plus-Noise-Ratio (SINR) model [8]. Although the global interference as defined in the physical SINR model poses a great challenge for finding efficient distributed solutions, algorithms designed under this model are easier to implement in practice.

Besides the interference issue, nodes in a newly formed wireless ad hoc network typically have no prior knowledge on the number of nodes in a proximity range. Although an estimate of the maximum number Δ of nodes' neighbors can be used to derive efficient randomized distributed local broadcasting algorithms as demonstrated in [6], it is shown in [2] that even in the graph based interference model, acquiring such an estimate may take more time than performing a local broadcast. So far, no efficient way to compute an estimate of Δ in the physical SINR model in a distributed setting has been proposed. Hence, a more practical choice is to derive efficient local broadcasting algorithms without information on neighborhood.

A. Our Results

In this work, under the practical physical interference model and assuming no information on neighborhood, we study the time complexity of distributively performing a local broadcast in unstructured wireless networks of which initially deployed wireless ad hoc and sensor networks are one kind (more details of this model can be found in Section II). We first propose a randomized distributed algorithm that completes the local broadcast in $O(\Delta \log n + \log^2 n)$ timeslots with high probability.[1] Compared with the state-of-the-art result

[1]We say "an event occurs with high probability" to mean that the event occurs with probability $1 - n^{-c}$ for a constant $c > 0$.

978-0-7695-4707-7/12 $26.00 © 2012 IEEE
DOI 10.1109/DCOSS.2012.39

in [22] under the same setting, our algorithm reduces the time complexity for all networks with non-constant Δ. For large Δ, e.g., $\Delta \in \Omega(\log n)$, our algorithm matches the algorithm in [6] which assumes each node knows the number of nodes in a proximity range. We also derive a new $\Omega(\Delta + \log n)$ time lower bound for the local broadcasting problem, which improves the previous trivial $\Omega(\Delta)$ result. This lower bound shows that our presented distributed algorithm achieves an $O(\log n)$ approximation ratio which represents a reduction by a logarithmic factor compared with the previous best result in [22] in terms of approximation ratio.

With reference to the algorithm in [22], we briefly explain how we achieve a faster result in this work. The algorithm in [22] employs a competition process to reduce the number of nodes that concurrently perform local broadcasting. In addition, nodes may need to iteratively execute the competition process $O(\Delta)$ times. In this work, to get a faster result, we adopt a clustering based strategy. Specifically, a set of leaders are first elected. These leaders will be responsible for coordinating the local broadcasting processes of their neighbors. The clustering based strategy guarantees that nodes participate in the competition process only once (to decide whether to become a leader or a non-leader). The main difficulty in implementing this clustering based strategy is that nodes have no knowledge about the neighborhood. Each non-leader needs to transmit a message to report to its leader about its existence. To overcome this difficulty, in absence of an estimate about Δ and under the asynchronous communication circumstance, we design a novel probability adjustment strategy for non-leaders and show that with this strategy, each non-leader can quickly send a message to its leader.

B. Related Work

The local broadcasting problem is closely related to the intensively studied broadcasting problem [14], the wake-up problem [5] and the contention resolution problem [4]. In the centralized setting, the local broadcasting problem, also called the Minimum-Latency Beaconing Schedule problem [20], has been very well studied. To the best of our knowledge, Alon et al. first studied the distributed local broadcasting problem in [1]. Assuming a synchronous circumstance and the existence of prior knowledge of Δ, they gave a randomized distributed algorithm which completes the local broadcasting in $O(\Delta \log n)$ rounds. Recently, by first computing an estimate of the local maximum degree for each node using $O(\Delta \log n + \log^2 n)$ time, Derbel and Talbi [2] generalized the above algorithm to the unknown neighborhood model. Both algorithms are derived under the graph based interference model. However, the message based fashion in estimating the local maximum degree in [2] is impossible to generalize to suit the physical interference model in which global interferences make deciding whether a message can be successfully received in a distributed setting difficult. Assuming the SINR model as in this work, Goussevaskaia et al. [6] first studied the local broadcasting problem. With the assumption that each node knows the number of nodes in its proximity region, their simple Aloha-like algorithm achieves a time complexity of $O(\Delta \log n)$, and then without this assumption, their randomized distributed algorithm uses $O(\Delta \log^3 n)$ time. The latter result was improved in a recent paper [22] by Yu et al., in which a distributed algorithm which completes the local broadcasting in $O(\Delta \log^2 n)$ time with high probability is given. By assuming that nodes can perform the physical carrier sensing, Yu et al. [22] also gave the first deterministic distributed local broadcasting algorithm with running time of $O(\Delta \log n)$.

The SINR model has received increasing attention since the seminal work [17] by Moscibroda and Wattenhofer. It has been shown that the network throughput can be increased significantly if and when the realistic SINR model is assumed [10], [18]. Goussevskaia et al. gave an excellent survey in [7] on approximation algorithms using the SINR model. However, the global interference as defined in the SINR model poses great challenges for designing distributed algorithms. Despite these challenges, there have been a few attempts in recent years. Assuming that all nodes can perform physical carrier sensing, Scheideler et al. [19] gave an $O(\log n)$ distributed algorithm for computing a constant approximate dominating set. Li et al. [16] presented a distributed algorithm for the minimum latency aggregation scheduling problem. Kesselheim and Vöcking [13] considered the contention resolution problem and showed that their distributed algorithm is asymptotically optimal up to a $\log^2 n$ factor. With a refined analysis, the approximation ratio of the algorithm in [13] was reduced to $O(\log n)$ in a recent paper [9], which was also shown to be the best possible for any distributed solution. Under the assumption that each node knows Δ, Derbel et al. [3] proposed a distributed coloring algorithm with $O(\Delta \log n)$ running time and $O(\Delta)$ colors. In [21], without requiring any knowledge on neighborhood, Yu et al. studied the $(\Delta+1)$-coloring in the physical model for the first time. Their proposed distributed $(\Delta+1)$-coloring algorithm achieves the same time complexity as the result in [3] for networks with $\Delta \in \Omega(\log n)$.

II. PROBLEM DEFINITIONS AND MODEL

Given a set of nodes V, the local broadcast range R is the distance up to which each node intends to broadcast its message. For each node v, the region within range R is denoted as B_v. A successful local broadcast of v is defined to be a transmission of a message such that it is successfully received by all wake-up nodes located in the local broadcasting region B_v. A local broadcast is complete if every node v in the network has transmitted a message to every other node in B_v. Given the local broadcast range R, the local broadcasting problem is to complete a local broadcast in the fewest timeslots.

For two nodes u and v, we denote $d(u, v)$ as the Euclidian distance between u and v. We say two nodes u and v are neighbors if they are within each other's local broadcast range, i.e., $d(u, v) \le R$. The neighborhood of a node v, i.e., the set of all its neighbors, is denoted as $N(v)$. For a node v, we denote by Δ_v the number of nodes in v's neighborhood. We write

$\Delta = \max_{v \in V} \Delta_v$. A set S of nodes is called an independent set if any two nodes of S are not in each other's neighborhood. An independent set S is maximal if any node not in S has a neighbor in S.

We depict the interference using the practical physical interference model (or the SINR model) [8]. In particular, a message sent by node u to node v can be correctly received at v iff

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{N + \sum_{w \in V \setminus \{u,v\}} \frac{P_w}{d(w,v)^\alpha}} \geq \beta, \tag{1}$$

where P_u (P_w) is the transmission power for node u (w); α is the path-loss exponent whose value is normally between 2 and 6; β is a hardware determined threshold value which is greater than 1; N is the ambient noise, and $\sum_{w \in V \setminus \{u,v\}} \frac{P_w}{d(w,v)^\alpha}$ is the interference experienced by the receiver v caused by all simultaneously transmitting nodes in the network.

As for the network communication model, the unstructured radio network model first presented in [15] is assumed in this work. Nodes are placed arbitrarily on the plane. They may wake up asynchronously without access to a global clock. A collision occurs at a node when multiple nodes transmit concurrently and none of these transmissions can be successfully decoded by the node. In this work, we do not assume there is collision detection functionality in the nodes, which means that nodes can not distinguish the cases of no transmission and a collision. Initially, any node has no information about the nodes in its close proximity, even the number of these nodes. The only prior knowledge given to nodes is a polynomial estimate n of the number of nodes in the network [11]. Furthermore, we assume that each node has a unique but arbitrary ID, which is only used for the receiver to identify the sender.

We apply the uniform power assignment, i.e., all nodes have the same transmission power level, which has been widely adopted in practice [7] due to its simplicity. The transmission range R_T of a node v is defined as the maximum distance at which a node u can receive a clear transmission from v ($SINR \geq \beta$) when there are no other simultaneous transmissions in the network. From the SINR condition (1), $R_T \leq R_{max} = (\frac{P}{\beta \cdot N})^{1/\alpha}$ for the given power level P. We further assume that $R_T < R_{max}$ and define $R_T = (P/cN\beta)^{1/\alpha}$, where $c > 1$ is a constant determined by the environment.

To simplify the analysis, we divide the time into timeslots that are synchronous among all the nodes. However, our algorithm does not rely on synchrony in any way. In each timeslot, a node can either send or keep silent. It can receive a message only if it wakes up and is not sending. In this work, due to the fact that nodes may wake up asynchronously, we define a node v's running time as the length of the interval from the timeslot when v starts executing the algorithm to the timeslot when v quits the algorithm, and the time complexity of the algorithm is the maximum of all nodes' running times. When synchronous waking-up is assumed, the above defined time complexity is just the longest time for any node to execute the algorithm.

III. Local Broadcasting Algorithm

In this section, we present the local broadcasting algorithm as given as Algorithm 1. In the algorithm, Greek letters represent constants. The basic idea of the algorithm is as follows. A Maximal Independent Set (MIS) in terms of the local broadcast range R is first computed, the nodes of which are called leaders. The leaders are responsible for arranging the local broadcasts of their neighbors. Each node not in the MIS chooses a leader in its neighborhood as its own leader. By receiving messages from other nodes, each leader acquires the knowledge of the nodes that it dominates. Then for each non-leader it dominates, a leader assigns a non-overlapping interval of timeslots such that the particular non-leader can accomplish the local broadcast quickly. In order to deal with newly wake-up nodes, each leader periodically transmits a message to inform newly waking-up nodes to start competing for the right to do local broadcast. To compute the MIS, we use the randomized distributed algorithm in [21] by which, with high probability, each node can decide whether to join the MIS in $O(\log^2 n)$ timeslots.

In Algorithm 1, we assign the transmission power as $P = cN\beta R^\alpha$. By the definition in Section II, the transmission range of nodes is R. There are three states in the algorithm: nodes in state $\mathcal{D}$ are leaders; nodes in state $\mathcal{C}$ are non-leaders that are competing for the right of local broadcasting; nodes in state $\mathcal{E}$ are non-leaders that are performing local broadcasting. There are also several controlling messages used in the algorithm. The $StartCompete$ message is used by a leader to inform non-leaders to compete for the right of local broadcasting, i.e., to make these non-leaders report their IDs such that the leader knows their existence. Non-leaders that received the $StartCompete$ message will join state $\mathcal{C}$. The $RequestRight$ message is transmitted by a non-leader to report its ID to its leader. The $Grant$ message is used by a leader to inform a particular non-leader to start performing local broadcasting. Non-leaders whose ID is contained in the $Grant$ message will join state $\mathcal{E}$. Apart from this function, the $Grant$ message is also used as a controlling message to adjust the transmission probability of nodes as shown in Algorithm 1. Next we describe the algorithm in more details.

After waking up, each node v first waits for $2\mu \log n$ timeslots (Line 1). During this period, if v received a $StartCompete_u$ message, it chooses u as its leader and joins state $\mathcal{C}$ (Lines 2–3). Otherwise, v starts executing the MIS algorithm as given in [21] (Line 4). After that, if v joins the MIS, it will execute different operations from the MIS algorithm in [21] in the last state $\mathcal{M}$ (which means the node joins the MIS). In the previous MIS algorithm in [21] that works as a subroutine of the coloring algorithm, after v joins state $\mathcal{M}$, with constant probability $2^{-\omega}$, v first transmits a waking-up message for $\mu \log n$ timeslots and then transmits a $DoNotTransmit$ message for also $\mu \log n$ timeslots. Here instead, with the same constant probability, a node v in state $\mathcal{M}$ will first perform a local broadcasting and then transmits a $StartCompete_v$ message, both for $\mu \log n$ timeslots, thus forcing all neighbors to join state $\mathcal{C}$. Using a similar analysis as in [21], it is easy to show that these messages can be successfully received by v's neighbors with high probability. Then v joins state $\mathcal{D}$. While node v stays in state $\mathcal{D}$, it adds each node u to a set Q_v that sends a $RequestRight_u(v)$ message to v (Line 10). If Q_v is not empty, it deletes the first node u from Q_v and transmits a $Grant_u(v)$ with constant probability for $\mu \log n$ timeslots (Lines 7–9), by which it gives u the right of performing a local broadcasting. In order to deal with asynchronously wake-ups, v takes at least $\mu \log n$ timeslots in every $2\mu \log n$ timeslots to transmit a $StartCompete_v$ message which informs newly waking-up nodes to join state $\mathcal{C}$ (Line 6).

For a non-leader u that stays in state $\mathcal{C}$, by continuously transmitting a $RequestRight_u$ message with probability p_u to report its ID to its leader v (Line 13), u competes for acquiring the right of local broadcasting. Because of the absence of knowledge of Δ, p_u is initially set as a small value determined by the parameter n (the size of the network). If u does not receive any $Grant$ message from its leader v for $3\mu \log n$ timeslots, which means that the nodes' transmission probabilities are not large enough to get a successful transmission, p_u is doubled (Line 12). However, in order to analyze Algorithm 1 and Algorithm MIS, we need to ensure that the sum of transmission probabilities in a local region can be bounded by a constant. The increase of transmission probability may make the sum in a local region exceed the bound. To avoid this, p_u will be halved if u receives a $Grant$ message from its leader v that is not for u (Line 15). After receiving a $Grant_u$ message from its leader v, u joins state $\mathcal{E}$ (Line 14), during which it performs a local broadcast with constant transmission probability for $\mu \log n$ timeslots (Lines 16–17).

In order to make sure that Algorithm 1 is correct with high probability, we set $\mu = \frac{9 \cdot 2^{\omega+4} 4^{3 \cdot 2^{1-\omega} \cdot \chi^{R_I+R,0.5R}}}{1-1/\rho}$ and $\omega = 6.4$, where ρ and R_I are constants as defined in Equation (3), and $\chi^{R_I+R,0.5R}$ is a also a constant as defined in Lemma 1 when R_I is determined. Furthermore, the value of ω in Algorithm 1 is determined by the MIS algorithm as given in [21].

A. Analysis

In this section, we show that with high probability, each node can perform a local broadcast after executing Algorithm 1 for $O(\Delta \log n + \log^2 n)$ timeslots. Before the analysis, we give some notations and definitions, the first of which is the definition of probabilistic interference which is the expected interference at the receiver.

Definition 1: For a node $v \in V$, the probabilistic interference at v, Ψ_v, is defined as the expected interference experienced by v in a certain timeslot t.

$$\Psi_v = \sum_{u \in V \setminus \{v\}} \frac{P_u p_u}{d(u,v)^\alpha}, \qquad (2)$$

where P_u is the transmission power and p_u is the sending probability of node u in timeslot t.

Algorithm 1 Local Broadcasting

Initially, $p_v = \frac{2^{-\omega}}{18n}; t_v = 0; Q_v = \emptyset; \omega = 6.4; leader = none$

Upon node v wakes up
1: wait for $2\mu \log n$ timeslots
2: **if** v received $StartCompete_u$ **then**
3: $state = \mathcal{C}; leader = u;$
4: **Else** execute the MIS algorithm
5: **end if**

Node v in state $\mathcal{D}$
6: **for** $\mu \log n$ timeslots **do** transmit $StartCompete_v$ with probability $2^{-\omega}$ **end for**
7: **if** Q_v is not empty **then**
8: **for** $\mu \log n$ timeslots **do** delete the first node u from Q_v and transmit $Grant_u(v)$ with probability $2^{-\omega}$ **end for**
9: **end if**

Message Received
10: **if** Received $RequestRight_u(v)$ **then** add u into Q_v **end if**

Node v in state $\mathcal{C}$
11: $t_v = t_v + 1$
12: **if** $t_v > 3\mu \log n$ **then** $p_v = 2p_v$; $t_v = 0$ **end if**
13: transmit $RequestRight_v(leader)$ with probability p_v;

Message Received
14: **if** received $Grant_v(leader)$ **then** $state = \mathcal{E}$ **end if**
15: **if** received $Grant_w(leader)$ for some other node w from v's leader that has not been received before **then** $p_v = p_v/2$; $t_v = 0$ **end if**

Node v in state $\mathcal{E}$
16: **for** $\mu \log n$ timeslots **do** $transmit()$ with probability $\frac{1}{9} \cdot 2^{-\omega}$ **end for**
17: quit;

A new parameter R_I is defined as follows,

$$R_I = R(2^{7-\omega} 3\sqrt{3}\pi\rho\beta \cdot \frac{1}{1-1/c} \cdot \frac{\alpha-1}{\alpha-2})^{1/(\alpha-2)}, \qquad (3)$$

where ρ is a constant larger than 1, such that $R_I > 2R$. In the following, we will show that the probabilistic interference caused by nodes with distance larger than R_I from the receiver can be bounded by a constant. Furthermore, Denote T_i, D_i and I_i as the disks centered at node i with radii R, $\frac{R}{2}$ and R_I, respectively. By E_i^r we denote the disk centered at node i with radius r. Without confusion, we also use T_i, D_i, I_i and E_i^r to denote the set of nodes in T_i, D_i, I_i and E_i^r, respectively. The following lemma is proved in [6] which will be used in the analysis.

Lemma 1 ([6]): Consider two disks D_1 and D_2 of radii R_1 and R_2, $R_1 > R_2$, we define χ^{R_1,R_2} to be the smallest number of disks D_2 needed to cover the larger disk D_1. It holds that $\chi^{R_1,R_2} \leq \frac{2\pi}{3\sqrt{3}} \cdot \frac{(R_1+2R_2)^2}{R_2^2}$.

We give two properties that can help bound the sum of transmission probabilities in any local region. These two properties are crucial for analyzing successful message transmissions. Property 1 has been shown to be correct with probability

$1 - O(n^{-2})$ in [21]. We will show that Property 2 is also true with high probability by Lemma 6.

Property 1: For any disk D_i and in any timeslot t throughout the execution of the algorithm, the sum of transmission probabilities of nodes in D_i that are executing the MIS algorithm is at most $3 \cdot 2^{-\omega}$.

Property 2: For any disk D_i and in any timeslot t throughout the execution of the algorithm,

(i) the sum of transmission probabilities of nodes in state $\mathcal{C}$ is at most $\sum_{u \in \mathcal{C}} \leq 2^{-\omega}$;

(ii) there is at most 9 nodes staying in state $\mathcal{E}$.

(iii) there is at most one node staying in state $\mathcal{D}$;

The following lemma is a direct corollary of Property 1 and Property 2 by noting the transmission probability assigned to nodes in states $\mathcal{E}$ and $\mathcal{D}$.

Lemma 2: Assume that Property 1 and Property 2 hold. For any disk D_i and in any timeslot t throughout the execution of the algorithm, the sum of transmission probabilities of nodes in D_i can be bounded as $\sum_{v \in D_i} p_v \leq 3 \cdot 2^{1-\omega}$.

In [21], it is shown that as long as the sum of transmission probabilities of nodes in any local region can be bounded by a constant, each node can correctly decide whether to join the MIS or not with high probability. Then based on Property 1 and Property 2, we state the correctness and the efficiency of the MIS algorithm in the following lemma. Since the proof for Lemma 3 is similar to that in [21], we omit it here for brevity.

Lemma 3: Assume that Property 1 and Property 2 hold. With probability $1 - O(n^{-2})$, every node $v \in V$ decides whether it should join the computed independent set or state $\mathcal{C}$ after executing the MIS algorithm for at most $O(\log^2 n)$ timeslots. And in any timeslot, the independent set computed by the MIS algorithm is correct with probability $1 - O(n^{-2})$.

In the subsequent analysis, we assume that in any timeslot, the independent set computed by the MIS algorithm is correct, and the error probability will be aggregated in the proof of the main theorem. Based on Lemma 2, we next give a sufficient condition for a successful transmission in the following lemma.

Lemma 4: Assume that Property 1 and Property 2 hold. If node v is the only sending node in $E_v^{R_I+R}$, with probability $1 - \frac{1}{\rho}$, the message sent by v will be received successfully by all nodes in T_v.

Proof: We first bound the probabilistic interference at a node $u \in T_v$ caused by nodes outside I_u.

Claim 1: For every node u, the probabilistic interference caused by nodes outside I_u can be bounded as: $\Psi_u^{w \notin I_u} \leq \frac{(1-1/c)P}{\rho \beta R^\alpha}$.

Proof: By Lemma 1 and Lemma 2, the sum of transmission probabilities in each T_i can be bounded as follows:

$$\sum_{x \in T_i} p_x \leq \frac{2\pi}{3\sqrt{3}} \cdot \frac{(R + 2 \cdot \frac{R}{2})^2}{(\frac{R}{2})^2} \cdot \sum_{x \in D_w} p_x \leq \frac{64\pi}{\sqrt{3} \cdot 2^\omega}. \quad (4)$$

Let $R_l = \{w \in V : lR_I \leq d(u, w) \leq (l+1)R_I\}$ and $\mathcal{I}$ be a maximum independent set in R_l. Clearly, $\mathcal{I}$ is also a dominating set in R_l. Thus $\sum_{i \in \mathcal{I}} T_i$ covers all the nodes in R_l. Furthermore, all disks D_i for every $i \in \mathcal{I}$ are mutually disjoint because of the independence of $\mathcal{I}$. Note that all these disks are located inside the extended region $R_l^+ = \{w \in V : lR_I - \frac{R}{2} \leq d(u,w) \leq (l+1)R_I + \frac{R}{2}\}$. Thus $|\mathcal{I}| \leq Area(R_l^+)/Area(disk(R/2))$. Then the probabilistic interference caused by nodes inside R_l is bounded as follows:

$$\begin{aligned}
\Psi_u^{R_l} &= \sum_{w \in R_l} \Psi_u^w \leq \frac{Area(R_l^+)}{Area(disk(R/2))} \cdot \max_{i \in I}\{\sum_{w \in T_i \cap R_l} \frac{P \cdot p_w}{(lR_I)^\alpha}\} \\
&\leq \frac{Area(R_l^+)}{Area(disk(R/2))} \cdot \frac{64\pi}{\sqrt{3} \cdot 2^\omega} \cdot \frac{P}{(lR_I)^\alpha} \\
&= \frac{\pi(((l+1)R_I + R/2)^2 - (lR_I - R/2)^2)}{\pi(R/2)^2} \\
&\quad \cdot \frac{64\pi}{\sqrt{3} \cdot 2^\omega} \cdot \frac{P}{(lR_I)^\alpha} \\
&= \frac{4(2l+1)(R_I^2 + R_I R)}{R^2} \cdot \frac{64\pi}{\sqrt{3} \cdot 2^\omega} \cdot \frac{P}{(lR_I)^\alpha} \\
&\leq \frac{1}{l^{\alpha-1}} \cdot \frac{9\pi \cdot 2^{7-\omega} P R_I^2}{\sqrt{3} R_I^\alpha R^2}.
\end{aligned}$$

The second inequality is by Inequality (4) and the last inequality is by $R < \frac{R_I}{2}$. Then

$$\begin{aligned}
\Psi_u^{w \notin I_u} &= \sum_{l=1}^{\infty} \Psi_u^{R_l} \leq \frac{9\pi \cdot 2^{7-\omega} P R_I^2}{\sqrt{3} R_I^\alpha R^2} \cdot \sum_{l=1}^{\infty} \frac{1}{l^{\alpha-1}} \\
&\leq \frac{9\pi \cdot 2^{7-\omega} P R_I^2}{\sqrt{3} R_I^\alpha R^2} \cdot \frac{\alpha - 1}{\alpha - 2} \\
&\leq \frac{(1-1/c)P}{\rho \beta R^\alpha}.
\end{aligned} \quad (5)$$

■

By the Markov inequality, with probability at least $1 - \frac{1}{\rho}$, the interference at a node u caused by nodes outside I_u can not exceed $\rho \Psi_u^{w \notin I_u}$. Then if v is the only sending node in $E_v^{R_I+R}$, i.e., v is the only sending node in I_u for every $u \in T_v$, by Lemma 1, with probability at least $1 - \frac{1}{\rho}$, the SINR at node u can be bounded as follows:

$$\frac{P/d(v,u)^\alpha}{N + \rho \Psi_u^{w \notin I_u}} \geq \frac{P/R^\alpha}{\frac{P}{c\beta R^\alpha} + \frac{(1-1/c)P}{\beta R^\alpha}} \geq \beta. \quad (6)$$

Thus u can successfully receive the message sent from v according to the SINR constraint (1). ■

Based on the sufficient condition given in Lemma 4, we list the successful transmissions of messages used in Algorithm 1.

Lemma 5: Assume that Property 1 and Property 2 hold. Then with probability at least $1 - \frac{1}{n^4}$, the following results are correct:

(i) A node v in state $\mathcal{D}$ can successfully send a message $Grant$ to all its neighbors in $\mu \log n$ timeslots.

(ii) A node v in state $\mathcal{D}$ can successfully send a message $StartCompete$ to all its neighbors in $\mu \log n$ timeslots.

(iii) A node v in state $\mathcal{E}$ can successfully perform a local broadcasting in $\mu \log n$ timeslots.

Proof: We only prove (i) here. (ii) and (iii) can be proved similarly.

As shown in Lemma 4, if v is the only sending node in $E_v^{R_I+R}$, with probability $1 - \frac{1}{\rho}$, the message $Grant$ sent by v

can be received successfully by all nodes in T_v. Let P_1 denote the event that v is the only sending node in $E_v^{R_I+R}$, then

$$
\begin{aligned}
P_1 &= 2^{-\omega} \prod_{u \in E_v^{R_I+R} \setminus \{v\}} (1-p_u) \\
&\geq 2^{-\omega} \prod_{u \in E_v^{R_I+R}} (1-p_u) \\
&\geq 2^{-\omega} \cdot (\frac{1}{4})^{\sum_{u \in E_v^{R_I+R}} p_u} \\
&\geq 2^{-\omega} \cdot (\frac{1}{4})^{3 \cdot 2^{1-\omega} \cdot \chi^{R_I+R, 0.5R}}.
\end{aligned} \tag{7}
$$

The last inequality is by Lemma 1 and Lemma 2. Then the probability P_{no} that v fails to transmit the message $Grant$ to all the nodes in T_v in $\mu \log n$ timeslots is at most

$$
\begin{aligned}
P_{no} &\leq (1-(1-1/\rho)2^{-\omega} \cdot (\frac{1}{4})^{3 \cdot 2^{1-\omega} \cdot \chi^{R_I+R,0.5R}})^{\mu \log n} \\
&\leq e^{-(1-1/\rho)2^{-\omega} \mu \log n \cdot (\frac{1}{4})^{3 \cdot 2^{1-\omega} \cdot \chi^{R_I+R,0.5R}}} \leq n^{-4}.
\end{aligned} \tag{8}
$$

Thus with probability $1-n^{-4}$, v can successfully transmit a $Grant$ message to all its neighbors in $\mu \log n$ timeslots. ■

In the following lemma, we show that Property 2 is true with high probability.

Lemma 6: With probability at least $1-O(n^{-2})$, none of Property 2 $(i)(ii)(iii)$ is the first one to be violated.

Proof: We first claim that each disk D_i can intersect with at most 9 disks that do not cover each other's centers. This can be easily proved by noting that the angle composed by lines between i and any pair of centers is at least 38°. Next we show that none of (i), (ii) and (iii) is the first property to be violated.

(i) Assume that (i) is the first property to be violated in timeslot t^* in disk D_i. Before t^*, we can still assume that all properties are correct. By Property 2 (iii), all nodes in state $\mathcal{D}$ are independent in terms of R. Because D_i can intersect with at most 9 disks where each pair of centers have distance larger than R, all nodes in D_i that are in state $\mathcal{C}$ belong to at most 9 leaders by timeslot t^*-1. Denote the set of these leaders as L, and for each node $v \in L$, denote $\mathcal{C}_v(t)$ as the set of nodes in state $\mathcal{C}$ that choose v as their leader in timeslot t. We claim that in timeslot t^*, all nodes in D_i that are in state $\mathcal{C}$ also belong to leaders in L. Otherwise, there is a new leader coming up in $E_i^{3R/2}$ in timeslot t^*, denoted as w. w must have distance less than R from another leader in L, denoted as u. Furthermore, w must have waken up before $t^* - \Omega(\log^2 n)$, since it needs $\Omega(\log^2 n)$ timeslots to execute the MIS algorithm as shown in [21]. By the algorithm, u will transmit $StartCompete$ message for $\mu \log n$ timeslots after joining the MIS, and when u is in state $\mathcal{D}$, it takes at least $\mu \log n$ timeslots to transmit $StartCompete$ in every $2\mu \log n$ timeslots. Also, by the algorithm, after joining the MIS and before entering state $\mathcal{D}$, u takes $2\mu \log n$ timeslots to perform the local broadcasting and transmits the $StartCompete$ message. So u must have joined the MIS since $t^* - 2\mu \log n - 1$. By Lemma 5 (ii), with probability $1-O(n^{-4})$, w must have received the $StartCompete$ message from u by t^*-1 and will not join state $\mathcal{D}$ in timeslot t^*. This contradiction shows that with probability at least $1-O(n^{-4})$, all nodes in D_i that are in state $\mathcal{C}$ are those in L. Next we show a slightly stronger result: with probability $1-O(n^{-3})$, for each leader v in L, the sum of transmission probabilities by nodes in $\mathcal{C}_v$ is at most $\frac{1}{9} \cdot 2^{-\omega}$ in any timeslot during the execution of the algorithm. Then there exists no such violating timeslot t^* for D_i with probability $1-O(n^{-3})$.

Otherwise, assume that in timeslot t, for the first time, $\sum_{u \in \mathcal{C}_v(t)} p_u > \frac{1}{9} \cdot 2^{-\omega}$. Denote $I = [t - 3\mu \log n, t)$. By Algorithm 1, every node in $\mathcal{C}$ doubles its transmission probability at most once during the interval. Furthermore, some newly waking-up nodes may join state $\mathcal{C}$ during the interval. However, the sum of transmission probabilities of newly joined nodes is at most $\frac{2^{-\omega}}{18n} \cdot n = \frac{1}{18} \cdot 2^{-\omega}$. Hence, it holds that in timeslot $t - 3\mu \log n$, the sum of transmission probabilities by nodes in $\mathcal{C}_v$ is at least $\frac{1}{36} \cdot 2^{-\omega}$. Consequently, before any violation timeslot, there is an interval I such that $\frac{1}{36} \cdot 2^{-\omega} \leq \sum_{u \in \mathcal{C}_v} p_u \leq \frac{1}{9} \cdot 2^{-\omega}$. Because Property 2 (i) is the first violated one, we can still assume that other properties are correct before t. So during the interval I, for any disk D_j, $j \neq i$, $\sum_{v \in D_j} p_v \leq 3 \cdot 2^{1-\omega}$. Next we show that with probability at least $1-n^{-4}$, v will successfully send a new $Grant$ message to all its neighbors during the interval $(t - 3\mu \log n, t)$. Clearly, if all nodes in $\mathcal{C}_v(t - 3\mu \log n)$ joins state $\mathcal{E}$ by $t-1$, then $\sum_{u \in C_{v1}(t)} p_u$ is at most the sum of transmission probabilities of newly joined nodes. As discussed above, it is at most $\frac{1}{18} \cdot 2^{-\omega}$. So in the following, it can be assumed that not all nodes in $\mathcal{C}_v(t - 3\mu \log n)$ have joined state $\mathcal{E}$ by time $t-1$.

We claim that at least one node in $\mathcal{C}_v$ can send a message $RequestRight$ to v during the interval $I_1 = [t - 3\mu \log n, t - 2\mu \log n - 1]$. Using a similar argument as in Lemma 4, if a node $w \in N(v)$ is the only transmitting node in $E_v^{R_I}$, then v can receive the message from w successfully with probability at least $1 - 1/\rho$. Denote D as a minimum cover of disks with radius $\frac{R}{2}$ for $E_v^{R_I}$. Then in any timeslot during I_1, the probability P_{only} that there is only one node $w \in \mathcal{C}_{v1}$ transmitting is

$$
\begin{aligned}
P_{only} &= \sum_{w \in \mathcal{C}_v} p_w \prod_{w' \in E_v^{R_I} \setminus \{w\}} (1-p_{w'}) \\
&\geq \sum_{w \in \mathcal{C}_v} p_w \prod_{D_j \in D} \prod_{w' \in D_j} (1-p_{w'}) \\
&\geq \sum_{w \in \mathcal{C}_v} p_w \prod_{D_j \in D} (\frac{1}{4})^{\sum_{w' \in D_j} p_{w'}} \\
&\geq \frac{1}{36} \cdot 2^{-\omega} \cdot (\frac{1}{4})^{\chi^{R_I, R/2} \cdot 3 \cdot 2^{1-\omega}}
\end{aligned} \tag{9}
$$

The last inequality is by Lemma 1 and Lemma 2. So during I_1, the probability P_T that there is not any node in $\mathcal{C}_v$ successfully transmitting a $RequestRight$ message to v is at most

$$
P_T \leq (1-(1-\frac{1}{\rho}) \cdot \frac{1}{36} \cdot 2^{-\omega} (\frac{1}{4})^{\chi^{R_I, R/2} \cdot 3 \cdot 2^{1-\omega}})^{\mu \log n} \leq n^{-4}. \tag{10}
$$

Thus with probability $1-n^{-4}$, v receives a $RequestRight$ message during the interval I_1. Denote t_1 as the first timeslot when v starts broadcasting a new $Grant$ message after $t - 3\mu \log n$. Because v broadcasts a $Grant$ message for $\mu \log n$ timeslots in every $2\mu \log n$ timeslots and v receives a new $RequestRight$ message by $t - 2\mu \log n$, such a timeslot exists in the interval $(t - 3\mu \log n, t - \mu \log n]$ with probability $1 - n^{-4}$. Then by Lemma 5 (ii), during the interval $(t - 3\mu \log n, t - 1]$, with probability $1 - n^{-4}$, all nodes in $\mathcal{C}_v$ receive a new $Grant_w$ message and halve their transmission probabilities except w which enters state $\mathcal{E}$. Denote t_2 as the first timeslot in which all nodes in $\mathcal{C}_v$ have successfully received the new $Grant$ message. So $t_2 \in (t - 3\mu \log n, t - 1]$. Then by Algorithm 1, all nodes in $\mathcal{C}_v$ will not increase the transmission probability until $t_2 + 3\mu \log n - (t_2 - t_1) = t_1 + 3\mu \log n > t$. Note that before halving the transmission probability, the sum of transmission probabilities of all nodes in $\mathcal{C}_v$ is at most $\frac{1}{9} \cdot 2^{-\omega}$. So during the interval $[t_2, t_1 + 3\mu \log n)$, the sum of transmission probabilities of these nodes is at most $\frac{1}{18} \cdot 2^{-\omega}$ with probability $1 - n^{-4}$. Also note that the sum of transmission probabilities of newly joined nodes is at most $\frac{1}{18} \cdot 2^{-\omega}$. Then during the interval $[t_2, t_1 + 3\mu \log n)$, $\sum_{u \in \mathcal{C}_v} p_u \leq \frac{1}{18} \cdot 2^{-\omega} + \frac{1}{18} \cdot 2^{-\omega} = \frac{1}{9} \cdot 2^{-\omega}$. Thus with probability $1 - n^{-4}$, D_i will not violate Property 2 (i) in timeslot t.

Finally, we bound the number of potential violating timeslots for v during the execution of the algorithm. From the above analysis, before each potential violation timeslot, there will be a node in $\mathcal{C}_v$ joining state $\mathcal{E}$. Thus there are at most n potential violating timeslots for v. So during the execution of the algorithm, with probability $1 - O(n^{-3})$, there exists no such violating timeslot for v. Then by the fact that non-leaders in D_i belong to at most 9 leaders, with probability at least $1 - O(n^{-3})$, Property 2 (i) is not the first violated one in D_i. And Property 2 (i) is not the first violated one for any disk with probability $1 - O(n^{-2})$.

(ii) Assume that (ii) is the first property to be violated, and D_i violates it for the first time in timeslot t. So there are at least 10 nodes staying in state $\mathcal{E}$. Using a similar analysis as in (i), we can show that there are at most 9 leaders within distance $3R/2$ from i in timeslot t. So there exist two nodes with the same leader. Denote these two nodes as u and v, and assume that u first joined state $\mathcal{E}$. We further assume that w is the leader of u and v.

Before t, we can still assume all properties hold, since Property 2 (ii) is the first one to be violated in timeslot t. By Algorithm 1, v joins state $\mathcal{E}$ after receiving a $Grant_v$ message from w. Clearly, w has started transmitting $Grant_v$ by the timeslot t. So w must have started transmitting the message $Grant_u$ from the timeslot $t - 2\mu \log n$. Then by Lemma 5 (i), u has received $Grant_u$ by the timeslot $t - \mu \log n - 1$ with probability $1 - O(n^{-4})$. By Algorithm 1, u stays in state $\mathcal{E}$ for $\mu \log n$ timeslots. Thus u will have quit the algorithm by timeslot $t - 1$. This contradiction shows that when u stays in state $\mathcal{E}$, D_i will not first violate Property 2 (ii) with probability $1 - O(n^{-4})$. So D_i is not the disk that first violates Property 2 (ii) with probability $1 - O(n^{-3})$. Then Property 2 (i) is not the first violated one for all disks with probability $1 - O(n^{-2})$.

(iii) Assume that (iii) is the first property to be violated in timeslot t in D_i. Assume that in timeslot t a node v joins state $\mathcal{D}$, and there is another node u also staying in state $\mathcal{D}$. As shown in [21], v needs to execute the MIS algorithm for $\Omega(\log^2 n)$ timeslots before joining state $\mathcal{D}$. Thus v must have waken up before $t - \Omega(\log^2 n)$. using a similar analysis as in **(i)** and by the fact that all properties are correct before t, we can obtain that v has joined state $\mathcal{C}$ before t with probability $1 - O(n^{-4})$. This contradiction shows that for node v, with probability at least $1 - O(n^{-4})$, it will not join state $\mathcal{D}$ if there has been a neighbor in state $\mathcal{D}$. Thus for D_i, Property 2 (iii) is not the first violated property with probability $1 - O(n^{-3})$. And Property 2 (iii) is not the first violated one for all disks with probability $1 - O(n^{-2})$. ■

Theorem 1: Each node v will correctly perform a local broadcast after waking up for $O(\Delta \log n + \log^2 n)$ timeslots with probability $1 - O(n^{-1})$.

Proof: We first prove that with probability $1 - O(n^{-2})$, a node v will correctly perform a local broadcast after waking up for $O(\Delta \log n + \log^2 n)$ timeslots. By Algorithm 1, after waking up, v first waits for $2\mu \log n$ timeslots, during which if v received a $StartCompete$ message, v will join state $\mathcal{C}$. Otherwise, v will start executing the MIS algorithm. By Lemma 3, after $O(\log^2 n)$ timeslots, v will join state $\mathcal{D}$ or $\mathcal{C}$ with probability $1 - O(n^{-2})$. Thus after waking up, v takes at most $O(\log^2 n)$ timeslots before entering state $\mathcal{D}$ or $\mathcal{C}$ with probability $1 - O(n^{-2})$. If v joins state $\mathcal{D}$, using a similar argument as for proving Lemma 5, we can prove that v has done a successful local broadcast after joining the MIS for $\mu \log n$ timeslots with probability $1 - O(n^{-4})$. If v joins state $\mathcal{C}$, by Algorithm 1 and Lemma 5 (iii), v will finally join state $\mathcal{E}$ and successfully perform a local broadcast during its stay in state $\mathcal{E}$ with probability $1 - O(n^{-4})$. From the algorithm, v stays in state $\mathcal{E}$ for $\mu \log n$ timeslots, so we only need to bound the number of timeslots that v stays in state $\mathcal{C}$.

By Algorithm 1, after joining state $\mathcal{C}$, if v does not receive any $Grant$ message from its leader u for $3\mu \log n$ timeslots, it doubles its transmission probability. If v received a new $Grant$ message not for v, v halves its transmission probability. Thus after at most $2(\Delta - 1) \times 3\mu \log n + \log n \times 3\mu \log n$ timeslots, either v has received a $Grant_v(u)$ message from its leader, or has a transmission probability of $\frac{1}{18} \cdot 2^{-\omega}$, since v can receive at most $(\Delta - 1)$ $Grant$ messages not for v from u. In the last case, using a similar analysis as in the proof of Lemma 5, we can show that v successfully transmits a $RequestRight_v(u)$ to node u with probability $1 - O(n^{-4})$ in the subsequent $2\mu \log n$ timeslots. By Algorithm 1 and Lemma 5, after at most $\Delta \cdot 2\mu \log n$ timeslots from then, v will receive a $Grant_v(u)$ message from u with probability at least $1 - O(n^{-4})$. Thus v stays in state $\mathcal{C}$ for at most $O(\Delta \log n + \log^2 n)$ timeslots with probability at least $1 - O(n^{-4})$.

Put all the above together, we now know that with probability $1 - O(n^{-2})$, v will successfully perform a local broadcast after waking up for $O(\Delta \log n + \log^2 n)$ timeslots. Note that the result is obtained based on the correctness of the MIS algorithm and under the assumption that Property 1 and

Property 2 hold. The correctness of the MIS algorithm is given in Lemma 3, and it has been shown that Property 1 is true with probability $1 - O(n^{-2})$. In Lemma 6, we also prove that Property 2 holds with probability $1 - O(n^{-2})$. So a node v can successfully perform a local broadcast after waking up for $O(\Delta \log n + \log^2 n)$ timeslots with probability $1 - O(n^{-2})$. This is true for every node with probability $1 - O(n^{-1})$, which completes the proof. ∎

IV. Lower Bound

Because each node can successfully receive at most one message in one timeslot, $\Omega(\Delta)$ is a natural lower bound for the local broadcasting problem. So for deriving the $\Omega(\Delta + \log n)$ lower bound, we only need to show that there exists a network with $\Delta \leq \log n$ such that any randomized algorithm needs $\Omega(\log n)$ timeslots to complete a local broadcast. We prove that this is true even in a synchronous circumstance.

Theorem 2: There exists a network with $\Delta = 1$ such that any randomized algorithm needs $\Omega(\log n)$ timeslots to perform a local broadcast with probability $1 - \frac{1}{n}$.

Proof: Consider a network where each node has only one neighbor in its transmission range (this neighbor is also within the local broadcast range). Before receiving a message from the neighbor, in a sequence of $\frac{\log n}{4}$ timeslots, the transmission probability of each node v can be calculated for each timeslot in advance. Because $n > 2^{\frac{\log n}{4}}$, there must exist two nodes u and v, such that in each timeslot, they both transmit with probability at least $\frac{1}{2}$ or less than $\frac{1}{2}$. We can construct the graph such that u and v are neighbors. Next we show that in the first $\frac{\log n}{4}$ timeslots, both u and v can not receive any message with high probability. Specifically, in a timeslot, the probability that u can successfully transmit a message to v or the other way around is at most $p_u(1-p_v)+p_v(1-p_u) \leq \frac{3}{4}$ for either case. Then the probability that u or v can successfully transmit a message after $\frac{\log n}{4}$ timeslots is at most $1-(\frac{1}{4})^{\frac{\log n}{4}} = 1-n^{-\frac{1}{2}}$. ∎

V. Conclusion

In this paper, we study the local broadcasting problem in unstructured wireless networks. Assuming the physical interference model, we propose a new randomized distributed algorithm with running time of $O(\Delta \log n + \log^2 n)$, which improves the state-of-the-art result in [22] for all networks with non-constant Δ. The proposed algorithm does not need any information of Δ. This feature makes our algorithm widely applicable, even in newly deployed networks. We have also derived a new lower bound for randomized distributed local broadcasting algorithms, which translates to an approximation ratio of $O(\log n)$ for our algorithm. This is a logarithmic factor reduction as compared with the previous best result in [22] in terms of approximation ratio. There are several interesting and meaningful directions for future work. The first direction is to derive a deterministic distributed algorithm for local broadcasting without physical carrier sensing. The second one is to study whether synchronous communication is useful for designing more efficient local broadcasting algorithms. Furthermore, as discussed in the related work, the local broadcast problem is closely related to many other problems. So it is also important to see whether the method for designing our algorithm can be used to obtain faster solutions for other problems, e.g., the wake-up problem [5], the multi-message broadcast problem [14] and the contention resolution problem [4].

VI. Acknowledgements

This work was supported in part by the National Basic Research Program of China Grant 2011CBA00300, 2011CBA00302, the National Natural Science Foundation of China Grant 61103186, 61073174, 61033001, 61061130540, the Hi-Tech research and Development Program of China Grant 2006AA10Z216, and Hong Kong RGC-GRF grants 714009E and 714311.

References

[1] N. Alon, A. Bar-Noy, N. Linial, D. Peleg. On the complexity of radio communication (Extended Abstract). In *STOC*, 1989.
[2] B. Derbel, E.-G. Talbi. Radio network distributed algorithms in the unknown neighborhood model. In *ICDCN*, 2010.
[3] B. Derbel, E.-G. Talbi. Distributed node coloring in the SINR model. In *ICDCS*, 2010.
[4] A. Fernández Anta, M. A. Mosteiro, J. R. Muñoz. Unbounded Contention Resolution in Multiple-Access Channels. In *DISC*, 2011.
[5] L. Gasieniec, A. Pelc, D. Peleg. The wakeup problem in synchronous broadcast systems. *SIAM J. Discrete Math.* 14(2):207-222, 2001.
[6] O. Goussevskaia, T. Moscibroda, R. Wattenhofer. Local broadcasting in the physical interference model. In *DialM-POMC*, 2008.
[7] O. Goussevskaia, Y. A. Pignolet, R. Wattenhofer. Efficiency of wireless networks: approximation algorithms for the physical interference model. *Foundations and Trends in Networking* 4(3): 313-420, 2010.
[8] P. Gupta, P. R. Kumar. The capacity of wireless networks. *IEEE Transaction on Infromation Theory*, Vol. 46, No. 2, pp. 388-404, 2000.
[9] M.M. Halldórsson, P. Mitra. Nearly Optimal Bounds for Distributed Wireless Scheduling in the SINR Model. In *ICALP*, 2011.
[10] Q.-S. Hua, F.C.M. Lau. Joint link scheduling and topology control for wireless sensor networks with SINR constraints. *Handbook of Research on Developments and Trends in Wireless Sensor Networks: From Principle to Practice*, IGI Global, pages 184-208, 2010.
[11] T. Jurdzinski, M. Kutylowski, J. Zatopianski. Energy-Efficient Size Approximation of Radio Networks with No Collision Detection. In *COCOON*, 2002.
[12] T. Jurdzinski, G. Stachowiak. Probabilistic algorithms for the wakeup problem in single-hop radio networks. In *ISAAC*, 2002.
[13] T. Kesselheim, B. Vöcking. Distributed Contention Resolution in Wireless Networks. In *DISC*, 2010.
[14] F. Kuhn, N.A. Lynch, C.C. Newport. The abstract MAC layer. *Distributed Computing* 24(3-4):187-206, 2011.
[15] F. Kuhn, T. Moscibroda, R. Wattenhofer. Initializing newly deployed Ad Hoc and sensor networks. In *MOBICOM*, 2004.
[16] H. Li, Q.-S. Hua, C. Wu, F.C.M. Lau. Minimum-latency aggregation scheduling in wireless sensor networks under physical interference model. In *MSWiM*, 2010.
[17] T. Moscibroda, R. Wattenhofer. The complexity of connectivity in wireless networks. In *INFOCOM*, 2006.
[18] T. Moscibroda, R. Wattenhofer, Y. Weber. Protocol design beyond graph-based models. In *HotNets*, 2006.
[19] C. Scheideler, A. Richa, P. Santi. An $O(logn)$ dominating set protocol for wireless ad-hoc networks under the physical interference model. In *Mobihoc*, 2008.
[20] P. Wan, X. Xu, L. Wang, X. Jia, E.K. Park. Minimum-latency beaconing schedule in multihop wireless networks. In *INFOCOM*, 2009.
[21] D. Yu, Y, Wang, Q.-S. Hua, and F.C.M. Lau. Distributed $(\Delta + 1)$-coloring in the physical model. In *ALGOSENSORS*, 2011.
[22] D. Yu, Y, Wang, Q.-S. Hua, and F.C.M. Lau. Distributed local broadcasting algorithms in the physical interference model. In *DCOSS*, 2011.