Distributed Local Broadcasting Algorithms in the Physical Interference Model

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Abstract—Given a set of sensor nodes V where each node
wants to broadcast a message to all its neighbors that are within
a certain broadcasting range, the local broadcasting problem is to
schedule all these requests in as few timeslots as possible. In this
paper, assuming the more realistic physical interference model
and no knowledge of the topology, we present three distributed
local broadcasting algorithms where the first one is for the
asynchronized model and the other two are for the synchronized
model. Under the asynchronized model, nodes may join the
execution of the protocol at any time and do not have access
to a global clock, for which we give a distributed randomized
algorithm with approximation ratio $O(\log^2 n)$. This improves the
state-of-the-art result given in [14] by a logarithmic factor. For
the synchronized model where communications among nodes are
synchronous and nodes can perform physical carrier sensing,
we propose two distributed deterministic local broadcasting
algorithms for synchronous and asynchronous node wakeups,
respectively. Both algorithms have approximation ratio $O(\log n)$.

I. INTRODUCTION

A. Problem Motivation

When a wireless ad hoc network begins to take shape, an
infrastructure must be established before any functions, such
as routing, can be performed. This can be done via local
broadcasting to neighboring nodes by the participating nodes.
Formally, given a set of sensor nodes V where each node
would like to broadcast a message to all its neighbors that
are within a certain broadcasting range, the local broadcasting
problem is to schedule all these requests in as few timeslots
as possible.

Unlike wired networks where a link is generally safe
and reliable, wireless radio communications are subject to
interference caused by simultaneous wireless transmissions.
The existence of interference is a major obstacle in the
design of efficient local broadcasting protocols. To study
the phenomenon, the two most widely adopted interference
models are the protocol (interference) model and the physical
(interference) model. In the protocol model, the interference
is modeled as a localized function whereby a node can hear
a message if there are no other simultaneously transmitting
senders within the receiver's exclusion region. The exclusion
region could be a disc centered at the receiver of which
the radius is defined by some distance function. However, in
reality, the interference experienced by a receiver is caused

not only by its nearby nodes that are in its exclusion region,
but also by nodes that are further away. In other words, the
"cumulative" interference caused by all the simultaneously
transmitting nodes, near and far, affects the reception of
any message by the receiver. The Signal-to-Interference-plus-
Noise-Ratio (SINR) model [17] tries to model this reality. The
SINR model assumes that a signal fades with the distance to
the power of some path-loss exponent α, and a signal can be
successfully decoded at the receiver iff the ratio of the received
signal strength and the sum of the interference caused by
nodes sending simultaneously plus noise is above a hardware-
defined threshold β. Since the SINR model defines a global
function, adopting this more realistic model makes the process
to develop efficient distributed algorithms more intricate.

In addition to the SINR model which deals with signal
transmissions and interferences, we need also to decide on
the communication model which is either asynchronized or
synchronized. In the asynchronized model, there is no global
clock, and sensor nodes can wake up asynchronously at
arbitrary times. In this paper, we assume that nodes wake up
spontaneously without being activated by some message. In
the synchronized model, there is a global clock accessible to
all nodes, and both asynchronous and synchronous node wake-
ups will be considered. We believe asynchronous wakeups are
more in line with the reality as it is hard to implement global
clocking in a large sensor network.

B. Our Contribution

This paper presents the following two results for the local
broadcasting problem under the physical interference model:

- For the asychronized model, we give a distributed ran-
  domized local broadcasting algorithm with approximation
  ratio $O(log^2 n)$. Compared to the current best algorithm
  which was proposed by Goussevskaia, Moscibroda and
  Wattenhofer in [14], our algorithm reduces the time
  complexity by a logarithmic factor.
- For the synchronized model where communications are
  carried out in rounds, we give two distributed determin-
  istic local broadcasting algorithms under the assumption
  that all nodes can perform physical carrier sensing. We
  consider both synchronous and asynchronous wakeups.

Our deterministic algorithms are optimal up to a logarithmic factor.

Our algorithms do not require any information about the topology, except an estimate of the number of nodes in the network.

C. Related Work

It has been shown that assuming the more realistic SINR model in a design could greatly increase the network throughput [19], [33]. Thus the SINR model has received increasing attention since the seminal work by Moscibroda and Wattenhofer in [32]. In [3], Brar et al. gave the first approximation algorithm for the link scheduling problem under uniform random node distributions. In [4], Chafekar et al. considered the so called cross-layer latency minimization problem that combines link scheduling and routing. In [20], Hua and Lau gave both exact and approximate link scheduling algorithms and they also considered how spread-spectrum techniques could affect the scheduling performance under the SINR model [21]. In [15], Goussevskaia et al. gave an excellent survey on approximation algorithms using the SINR model. More recent works on using the SINR model can be found in [2], [9], [11], [12], [18], [19], [23], [29], [31].

Despite the difficulties, there have been some attempts to design distributed algorithms under the SINR model. Li et al. [28] gave the first distributed algorithm for the minimum latency aggregation scheduling problem. Derbel et al. [8] proposed a distributed node coloring algorithm with asynchronous node wakeups. Scheideler et al. [35] presented a distributed approximation algorithm for the dominating set problem. They all assume the SINR model.

As a building block, local broadcasting has been frequently used in various broadcasting scenarios where a source has a message to send to all other nodes in the network [26], [27], [36]. The local broadcasting problem is also closely related to the intensively studied wake-up problem [5], [6], [10], [13], [24], [25] in which nodes that have waken would wake up their neighbors by sending them a message. For the local broadcasting operation itself, to our knowledge, the *single-round simulation* (SRS) primitive proposed by Alon et al. in [1] can be regarded as the first distributed technique. However, these SRS algorithms require all nodes start executing the algorithm at the same time and assume a synchronous environment. Derbel and Talbi [7] adapted the randomized protocol in [1] to the unknown neighborhood model where no information about node degrees is available. Closely related work to what we present here include two randomized algorithms for the local broadcasting problem proposed by Goussevaskaia et al. in [14]. With the assumption that each node knows the number of nodes in its proximity region, their simple Aloha-like algorithm achieves an approximation ratio of $O(\log n)$, and then without this assumption, their randomized distributed algorithm achieves an approximation ratio of $O(\log^3 n)$. Both algorithms considered asynchronous wakeups and used the SINR model. In contrast, all our algorithms do not assume

each node knows the number of nodes in its proximity region, which should be more reasonable in real deployment.

D. Structure

The remainder of the paper is organized as follows. We present the problem model in Section II. We give the distributed randomized local broadcasting algorithm for the asynchronized model in Section III, and the distributed deterministic local broadcasting algorithms for the synchronized model and for both synchronous and asynchronos wakeups in Section IV. We conclude the paper in Section V.

II. MODEL

We assume that nodes are placed arbitrarily on the plane. Given two nodes u and v, we denote by $d(u, v)$ the Euclidian distance between u and v. As mentioned in the introduction, by assuming the SINR model, a message sent by node u to node v can be correctly received at v iff

$$\frac{\frac{P}{d(u,v)^\alpha}}{N + \sum_{w \in V \setminus \{u,v\}} \frac{P}{d(w,v)^\alpha}} \geq \beta, \tag{1}$$

Here P is the transmission power of each node and this kind of uniform power assignment has been widely adopted in practice [16]; α is the path-loss exponent whose value is normally between 2 and 6; β is a hardware determined threshold value which is greater than 1; N is the ambient noise; $\sum_{w \in V \setminus \{u,v\}} \frac{P}{d(w,v)^\alpha}$ is the interference experienced by the receiver v, which is caused by all the simultaneously transmitting nodes in the network.

Given a set of nodes V, the local broadcast range R_B of a node $v \in V$ is the distance up to which v intends to broadcast its message. The region within this range and the number of nodes in it are denoted as B_v and Δ_v^B, respectively. For each node v, a successful local broadcast is defined to be a transmission of a message, such that it is successfully received by all nodes located in the local broadcasting region B_v under the SINR condition (1). A local broadcast is complete if every node v in the network has transmitted a message to every other node in B_v. Given the local broadcast range R_B, the local broadcasting problem is to complete a local broadcast in as few timeslots as possible.

In the absence of any other simultaneously transmissions in the network, the transmission range R_T of a node v is the maximum distance at which a node u can receive a clear transmission from v ($SINR \geq \beta$). The SINR condition (1) tells us that $R_T \leq (\frac{P}{\beta \cdot N})^{1/\alpha}$ for given power level P. We refer to the region within this range and the number of nodes in it as T_v and Δ_v^T, respectively.

We consider the local broadcasting problem for the unknown neighborhood model, i.e., nodes are clueless about the number of nodes in its close proximity with which they have to compete for the shared medium. In practice, the number of nodes in a network may not be known exactly, but it can be roughly estimated in advance. Here we assume the estimate of the total number of nodes in the network is an upper bound of a real number, i.e., $\hat{n} = n^c$ for some constant $c \geq 1$. Both

synchronized and asynchronized communication models are considered in this work.

In the synchronized scenario in which communication among nodes is done in synchronized rounds, we assume that all nodes have unique IDs from the interval $[1, n]$ using the same number of bits, i.e., small IDs have a prefix filled with 0s such that all IDs have equal length, where n is the estimate of the number of nodes. A polynomial bounded estimate of the number of nodes will not affect the time bounds of the proposed algorithms. Also, synchronous beginning is not assumed, i.e., each node may start the execution of the algorithm at an unknown point in time. In this case, the time complexity of an algorithm is defined as the number of rounds from the time that the last node starts to execute the algorithm until all nodes have performed a successful local broadcast. In every round, a node v can either listen or transmit. Furthermore, we assume that listening nodes can perform physical carrier sensing by energy detection.

Physical carrier sensing is part of the IEEE 802.11 standard, and is provided by a Clear Channel Assessment (CCA) circuit [35]. Given a certain sensing threshold T, a node v senses a busy channel if and only if it can sense a power larger than T. We do not assume that nodes can adjust their carrier sensing threshold during the execution of the algorithm. Instead, we adopt the concept of a carrier sensing range as introduced in many studies of 802.11 networks. The carrier sensing range R_S [11] is mapped from the carrier sensing power threshold T:

$$R_S = (\frac{P}{T})^{\frac{1}{\alpha}}, \tag{2}$$

where P is the transmission power. Consider a node v and a transmitter u. v can carrier-sense u if and only if the distance between v and u is not larger than R_S, i.e.,

$$d(u, v) \leq R_S. \tag{3}$$

In the asynchronized scenario, we assume a particularly harsh model of computation, which closely reflects the deployment of many real ad hoc and sensor systems. In particular, we assume that nodes may start executing the algorithm at any time and do not have access to a global clock. The only a-prior knowledge given to the nodes is an estimate of the number of nodes in the network. Furthermore, all nodes cannot perform physical carrier sensing. For ease of analysis, we divide the time into timeslots that are synchronized among all nodes. However, our algorithm does not rely on synchronization in any way. By some standard argument introduced in [34] for slotted vs. unslotted ALOHA, the realistic unslotted case differs only by a factor of two from the idealized slotted case. In this scenario, a node v's running time is defined as the length of the interval from the timeslot when v starts executing the algorithm to the timeslot when v quits from the algorithm. The time complexity of the algorithm is the maximum of all nodes' running times.

In this paper, we use the term "with high probability" to denote that an event that occurs with probability $1 - n^{-c}$ for a constant $c > 0$. We conclude this section with some useful facts and a lemma. Facts 1 and 2 can be found in many mathematical textbooks, and Lemma 1 is proved in [14].

Fact 1: Given a set of probabilities $p_1, \ldots, p_n$ with $\forall i : p_i \in [0, \frac{1}{2}]$, the following inequalities hold:

$$(1/4)^{\sum_{k=1}^n p_k} \leq \prod_{k=1}^n (1 - p_k) \leq (1/e)^{\sum_{k=1}^n p_k}. \tag{4}$$

Fact 2: For all n, t, with $n \geq 1$ and $|t| \leq n$, it holds that

$$e^t (1 - \frac{t^2}{n}) \leq (1 + t/n)^n \leq e^t. \tag{5}$$

Lemma 1 ([14]): Consider two discs D_1 and D_2 of radii R_1 and R_2, respectively, $R_1 > R_2$, we define χ^{R_1, R_2} to be the smallest number of discs D_2 needed to cover the larger disc D_1. It holds that

$$\chi^{R_1, R_2} \leq \frac{2\pi}{3\sqrt{3}} \cdot \frac{(R_1 + 2R_2)^2}{R_2^2}. \tag{6}$$

III. RANDOMIZED ALGORITHM IN ASYNCHRONIZED MODEL

A. Algorithm

In this section, we present a randomized local broadcasting algorithm—Algorithm 1. Every node transmits with power level $P = 2N\beta R_B^\alpha$. Greek letters represent constants. The algorithm is designed such that in any timeslot, the number of nodes simultaneously transmitting in a certain area of the network is small enough to permit each node to perform a successful local broadcast in $O(\log^2 n)$ timeslots with high probability. The following gives more details.

At the beginning of the algorithm's execution, a node is in the *waiting state* $\mathcal{W}$ in which it only listens. The purpose of state $\mathcal{W}$ is that newly joining or restarting nodes will not interfere with nodes that are carrying out the local broadcasting. In the waiting state, a node listens for messages and increases its counter $step$ in each timeslot. Once the $step$ counter of a node $v \in \mathcal{W}$ reaches the threshold, it joins the *active state* $\mathcal{A}$. An active node v tries to join state $\mathcal{B}$ in a certain timeslot by increasing its probability q_v for sending a message m_A. Initially, an active node v starts with a small probability q_v, and then doubles q_v every $\lambda \log n$ timeslots, and thus it exponentially increases its chance to join state $\mathcal{B}$. The design of state $\mathcal{A}$ is to bound the number of nodes simultaneously transmitting in state $\mathcal{B}$ in a certain area of the network. If an active node $v \in \mathcal{A}$ receives a message m_A from another active node, it will restart the algorithm, i.e., set its state to $\mathcal{W}$ and counter $step$ to 0. Once an active node $v \in \mathcal{A}$ sends a message m_A, it joins state $\mathcal{B}$ and performs a local broadcast in $\delta \log^2 n$ timeslots with a specified sending probability.

In order to guarantee the correctness of the algorithm and to obtain the high probability results in Section III-B, we set the constant parameters as follows: $\omega = 6.4$, $\delta = \frac{240e \cdot 2^\omega}{(1 - \frac{1}{\rho})^2} \cdot 4^{2^{2-\omega} \chi^{R_I + R_B, 0.5 R_B}}$, $\lambda = \frac{2^{\omega + 4} \cdot 4^{\frac{\frac{5}{4} + 2\chi^{R_I + 0.5 R_B, 0.5 R_B}}{2^\omega}}}{1 - \frac{1}{\rho}}$, $\kappa = \frac{120e \cdot 4^{2^{1-\omega} \chi^{R_I + 0.5 R_B, 0.5 R_B}}}{1 - 1/\rho}$, $\tau = \kappa^{-1}$. Here ρ is a constant defined in Section III-B and R_I (defined in Equation (7)) is also a constant as long as the transmission power P is given.

Algorithm 1 Asynchronized Local Broadcasting Algorithm

Upon node v wake-up:
1: $step = 0; state = \mathcal{W}; q_v = \frac{2^{-\omega-1}}{n}; q_B = \frac{\tau}{2^\omega \log n}; P = 2N\beta R_B^\alpha$

node v in state $\mathcal{W}$
1: **while** $state = \mathcal{W}$ **do**
2:    $step := step + 1$
3:    **if** $step \geq \delta \log^2 n$ **then**
4:       $state := \mathcal{A}; step := 0;$
5:    **end if**
6:    **if** m_A received **then**
7:       $step := 0;$
8:    **end if**

node v in state $\mathcal{A}$
1: **while** $state = \mathcal{A}$ **do**
2:    $step := step + 1$
3:    **if** $step \geq \lambda \log n$ **then**
4:       $q_v = 2q_v; step = 0;$
5:    **end if**
6:    $s := \begin{cases} 1, & with\ probability\ q_v \\ 0, & with\ probability\ 1 - q_v \end{cases}$
7:    **if** $s = 1$ **then**
8:       send(m_A) with power P; $state = \mathcal{B}$;
9:    **end if**
10:   **if** m_A received **then**
11:      $state := \mathcal{W}; step := 0;$
12:   **end if**

node v in state $\mathcal{B}$
1: **while** $state = \mathcal{B}$ **do**
2:    **for** $\delta \log^2 n$ time slots **do**
3:       $transmit()$ with probability q_B and power P;
4:    **end for**
5: quit from the algorithm

B. Analysis

In this section, let

$$R_I = R_B (2^{8-\omega}\sqrt{3}\rho\beta \cdot \frac{\alpha-1}{\alpha-2})^{1/(\alpha-2)}, \quad (7)$$

where ρ is a constant larger than 1. Here ρ is chosen such that $R_I > 2R_B$. Denote B_i, D_i and I_i as the discs centered at node i with radius R_B, $\frac{R_B}{2}$ and R_I, respectively. By E_i^r we denote the disc centered at node i with radius r. By $\mathcal{A}_i$ we denote the set of nodes in state $\mathcal{A}$ in disc D_i. $\mathcal{B}_i$ and $\mathcal{W}_i$ are defined similarly. Without confusion, we also use B_i, D_i, I_i and E_i^r to denote the set of nodes in B_i, D_i, I_i and E_i^r, respectively. Next we use the concept of probabilistic interference introduced in [14] in our analysis.

Definition 1: For a node $v \in V$, the probabilistic interference at v, Ψ_v, is defined as the expected interference experienced by v in a certain timeslot t.

$$\Psi_v = P \sum_{u \in V \setminus \{v\}} \frac{p_u}{d(u,v)^\alpha}, \quad (8)$$

where P is the transmission power and p_u is the sending probability of node u in timeslot t.

We begin the analysis by firstly presenting two properties. The first property states that the sum of sending probabilities by active nodes is bounded by a constant, and the second property states that the number of simultaneously broadcasting nodes is bounded.

Property 1: For all discs D_i and at any timeslot t throughout the execution of the algorithm, $\sum_{v \in A_i} q_v(t) \leq 2^{-\omega}$.

Property 2: For all discs D_i and at any timeslot t throughout the execution of the algorithm, $|\mathcal{B}_i| \leq \kappa \log n$.

We adopt a similar idea as that used in [30] to show that with high probability, neither property 1 nor property 2 is the first one to be violated during the execution of Algorithm 1. Note that our analysis is based on the SINR model, which is challenging. Due to the lack of space, the detailed proof of the following lemma can be found in the full version [37].

Lemma 2: During the execution of Algorithm 1, Properties 1 and 2 hold with probability at least $1 - O(n^{-2})$.

In the following, we use p_v to denote the sending probability of node v, i.e., $p_v = q_v$ if v is in state $\mathcal{A}$ and $p_v = q_B$ if v is in state $\mathcal{B}$.

Lemma 3: Assume Properties 1 and 2 hold. Then for every node u, the probabilistic interference caused by nodes outside I_u can be bounded by: $\Psi_u^{v \notin I_u} \leq \frac{P}{2\rho\beta R_B^\alpha}$.

Proof: We first bound the sum of transmission probabilities in each disc D_i. Note that nodes in $\mathcal{W}$ do not send. Based on the assumption that Properties 1 and 2 hold, we have

$$\begin{aligned} \sum_{v \in D_i} p_v &= \sum_{v \in \mathcal{A}_i} q_v + \sum_{v \in \mathcal{B}_i} q_B \\ &\leq 2^{-\omega} + \kappa \log n \cdot \frac{\tau}{2^\omega \log n} \\ &= 2^{1-\omega}. \end{aligned} \quad (9)$$

Then the sum of transmission probabilities in each B_i can be bounded as follows:

$$\begin{aligned} \sum_{v \in B_i} p_v &\leq \frac{2\pi}{3\sqrt{3}} \cdot \frac{(R_B + 2 \cdot \frac{R_B}{2})^2}{(\frac{R_B}{2})^2} \cdot \sum_{v \in D_w} p_v \\ &\leq \frac{64\pi}{3\sqrt{3} \cdot 2^\omega}. \end{aligned} \quad (10)$$

The first inequality is by Lemma 1. Let $R_l = \{v \in V : lR_I \leq d(u,v) \leq (l+1)R_I\}$. Let $\mathcal{I}$ be a maximum independent set in R_l. Clearly, $\mathcal{I}$ is also a dominating set in R_l. Thus $\sum_{v \in \mathcal{I}} B_v$ covers all nodes in R_l. Furthermore, all discs D_i for every $i \in \mathcal{I}$ are mutually disjoint because of the independence of $\mathcal{I}$ and $R_T = 2^{\frac{1}{\alpha}} R_B > R_B$. Note that all these discs are located inside the extended region $R_l^+ = \{v \in V : lR_I - \frac{R_B}{2} \leq d(u,v) \leq (l+1)R_I + \frac{R_B}{2}\}$. Thus $|\mathcal{I}| \leq Area(R_l^+)/Area(Disc(R_B/2))$. Then the probabilistic

interference caused by nodes inside R_l is bounded as follows:

$$\Psi_u^{R_l} = \sum_{v \in R_l} \Psi_u^v$$
$$\leq \frac{Area(R_l^+)}{Area(Disc(R_B/2))} \cdot \max_{i \in I} \{ \sum_{v \in B_i \cap R_l} \frac{P \cdot p_v}{(lR_I)^\alpha} \}$$
$$\leq \frac{Area(R_l^+)}{Area(Disc(R_B/2))} \cdot \frac{64\pi}{3\sqrt{3} \cdot 2^\omega} \cdot \frac{P}{(lR_I)^\alpha}$$
$$= \frac{\pi(((l+1)R_I + R_B/2)^2 - (lR_I - R_B/2)^2)}{\pi(R_B/2)^2} \quad (11)$$
$$\cdot \frac{64\pi}{3\sqrt{3} \cdot 2^\omega} \cdot \frac{P}{(lR_I)^\alpha}$$
$$= \frac{4(2l+1)(R_I^2 + R_I R_B)}{R_B^2} \cdot \frac{64\pi}{3\sqrt{3} \cdot 2^\omega} \cdot \frac{P}{(lR_I)^\alpha}$$
$$\leq \frac{1}{l^{\alpha-1}} \cdot \frac{384 P R_I^2}{\sqrt{3} \cdot 2^\omega R_I^\alpha R_B^2}.$$

The second inequality is by Equation (10) and the last inequality is by $R_B < \frac{R_I}{2}$. Then

$$\Psi_u^{v \notin I_u} = \sum_{l=1}^{\infty} \Psi_u^{R_l} \leq \frac{384 P R_I^2}{\sqrt{3} \cdot 2^\omega R_I^\alpha R_B^2} \cdot \sum_{l=1}^{\infty} \frac{1}{l^{\alpha-1}}$$
$$\leq \frac{384 P R_I^2}{\sqrt{3} \cdot 2^\omega R_I^\alpha R_B^2} \cdot \frac{\alpha-1}{\alpha-2} \quad (12)$$
$$\leq \frac{P}{2\rho\beta R_B^\alpha}.$$

∎

Lemma 4: Assume Properties 1 and 2 hold. If node v is the only sending node in $E_v^{R_I + R_B}$, with probability $1 - \frac{1}{\rho}$, the message sent by v will be received successfully by all nodes in B_v.

Proof: By the Markov inequality, with probability at least $1 - \frac{1}{\rho}$, the interference at some node u caused by nodes outside I_u cannot exceed $\rho\Psi_u^{v \notin I_u}$. Then if v is the only sending node in $E_v^{R_I + R_B}$, i.e., v is the only sending node in I_u for every $u \in B_v$, by Lemma 3, with probability at least $1 - \frac{1}{\rho}$, the SINR at node u can be bounded as follows:

$$\frac{\frac{P}{d(u,v)^\alpha}}{\rho\Psi_u^{w \notin I_u} + N} \geq \frac{\frac{P}{R_B^\alpha}}{\frac{P}{2\beta R_B^\alpha} + \frac{P}{2\beta R_B^\alpha}} \geq \beta \quad (13)$$

The claim holds. ∎

Lemma 5: Assume that Properties 1 and 2 hold. Let t_v be a timeslot in which a node v joins state $\mathcal{B}$. Then v can successfully perform a local broadcast in the subsequent $\delta \log^2 n$ timeslots with probability $1 - O(n^{-2})$.

Proof: Let P_1 be the probability that v is the only sending node in $E_i^{R_I + R_B}$. Then

$$P_1 = q_B \cdot \prod_{u \in E_v^{R_I + R_B} \setminus \{v\}} (1 - p_u)$$
$$\geq q_B \cdot \prod_{u \in E_v^{R_I + R_B}} (1 - p_u) \quad (14)$$
$$\geq q_B \cdot (\frac{1}{4})^{\sum_{u \in E_v^{R_I + R_B}} p_u}$$

The last inequality is by Fact 1. By Lemma 1 and Equation (9), we have

$$\sum_{u \in E_i^{R_I + R_B}} p_u \leq 2^{1-\omega} \chi^{R_I + R_B, 0.5 R_B}. \quad (15)$$

Thus $P_1 \geq \frac{\tau}{2^\omega \log n} \cdot (\frac{1}{4})^{2^{1-\omega} \chi^{R_I + R_B, 0.5 R_B}}$. By Lemma 4, we know that v will successfully perform a local broadcast with probability at least $1 - 1/\rho$ if v is the only sending node in $E_v^{R_I + R_B}$. Let P_{no} be the probability that v cannot perform a local broadcast within $\delta \log^2 n$ timeslots from t_v. Then

$$P_{no} \leq (1 - (1 - \frac{1}{\rho}) \cdot \frac{\tau}{2^\omega \log n} \cdot (\frac{1}{4})^{2^{1-\omega} \chi^{R_I + R_B, 0.5 R_B}})^{\delta \log^2 n}$$
$$\leq e^{-\delta \log^2 n \cdot (1 - \frac{1}{\rho}) \cdot \frac{\tau}{2^\omega \log n} \cdot (\frac{1}{4})^{2^{1-\omega} \chi^{R_I + R_B, 0.5 R_B}}} \in O(n^{-2}). \quad (16)$$

∎

Lemma 6: Let Δ_v^T be the number of nodes in v's transmission region. Then v can perform a local broadcast after $O(\Delta_v^T \log^2 n)$ timeslots with probability at least $1 - O(n^{-2})$. The bound holds for all nodes with probability at least $1 - O(n^{-1})$.

Proof: For a given v, let $\mathcal{T}_\mathcal{W}$, $\mathcal{T}_\mathcal{A}$ and $\mathcal{T}_\mathcal{B}$ be the total time v spends in the corresponding state during the execution of Algorithm 1. From Algorithm 1, we know that $\mathcal{T}_\mathcal{B} = \delta \log^2 n$. Under the assumption that Properties 1 and 2 hold, by Lemma 5, with probability at least $1 - O(n^{-2})$, v will successfully perform a local broadcast in $\delta \log^2 n$ timeslots after joining state $\mathcal{B}$. What remains is to bound $\mathcal{T}_\mathcal{W}$ and $\mathcal{T}_\mathcal{A}$.

If v does not receive a message m_A from a neighbor node for $\delta \log^2 n$ timeslots after entering state $\mathcal{W}$, it will join state $\mathcal{A}$. Unless it receives a message m_A, its sending probability will increase to $2^{-\omega-2}$ after $(\log n - 1)\lambda \log n$ timeslots. If v still does not receive m_A in the following $\lambda \log n$ timeslots, the probability that v does not send during these $\lambda \log n$ timeslots is at most $(1 - 2^{-\omega-2})^{\lambda \log n} \in O(n^{-3})$.

Thus it holds that after at most $\delta \log^2 n + \lambda \log^2 n$ timeslots, a node w in v's transmission region will enter state $\mathcal{B}$ with probability at least $1 - O(n^{-3})$. Since there are at most $\Delta_v^T - 1$ nodes in v's neighborhood, after at most $(\Delta_v^T - 1)(\delta \log^2 n + \lambda \log^2 n)$ timeslots, there will be no node preventing v entering $\mathcal{B}$ with probability at least $1 - O(n^{-2})$, and in the subsequent $\delta \log^2 n + \lambda \log^2 n$ timeslots, v will enter state $\mathcal{B}$ with probability $1 - O(n^{-3})$ as shown before. Thus with probability at least $1 - O(n^{-2})$, $\mathcal{T}_\mathcal{W} + \mathcal{T}_\mathcal{A} \leq \Delta_v^T(\delta \log^2 n + \lambda \log^2 n) \in O(\Delta_v^T \log^2 n)$. For all nodes, this is true with probability at least $1 - O(n^{-1})$. Finally, Properties 1 and 2 hold with probability at least $1 - O(n^{-2})$, which concludes the proof. ∎

Based on a trivial running time lower bound for local broadcasting algorithms, we can evaluate the performance of Algorithm 1 as follows. Note that the trivial lower bound had appeared in [14].

Theorem 1: Compared to the optimal solution for local broadcasting, Algorithm 1 can achieve an approximation ratio of $O(\log^2 n)$ in time complexity.

Proof: Denote Δ^B as the maximum number of nodes in the broadcasting region of any node, i.e., $\Delta^B = \max\{\Delta_v^B : v \in V\}$. When all nodes successfully perform local broadcasting, nodes in the broadcasting region of any node will correctly receive its message. In other words, each node will successfully receive messages from all nodes in its broadcasting region after the local broadcasting algorithm terminates. Hence, Δ^B is a trivial lower bound for the running time of any local broadcasting algorithm, since the receiver can decode the signal of only one sender at a time.

Let $\Delta^T = \max\{\Delta_v^T : v \in V\}$. On one hand, by Lemma 6, the time complexity of Algorithm 1 is $O(\Delta^T \log^2 n)$. On the other hand, by Lemma 1, $\Delta^T \leq \chi^{R_T,R_B} \cdot \Delta^B \in O(\Delta^B)$. Then the approximation ratio can be obtained. ∎

Remark 1: In Algorithm 1, we assign the transmission power as $P = 2N\beta R_B^\alpha$, which is smaller by a 2^α factor when compared to that in the state-of-the-art result in [14]. Hence, our algorithm is not only faster by a logarithmic factor than the previous result in [14], but also more efficient in energy consumption.

IV. DETERMINISTIC ALGORITHMS IN SYNCHRONIZED MODEL

In this section, we assume that communications among nodes are done in synchronized rounds, and all nodes have unique IDs from the interval $[1, n]$ using the same number of bits, where n is an estimate of the number of nodes in the network. Furthermore, all nodes can perform physical carrier sensing. We consider both the cases that nodes wake up and start executing the algorithm synchronously and asynchronously.

A. Unit Disc Graph Model And Collision Detection Model

The unit disc graph model (UDG) is a classic theoretical model for wireless networks, in which nodes having omnidirectional radio antennas are deployed in a planar, unobstructed environment. There exists an edge (communication link) between two nodes u and v if and only if their Euclidean distance is at most 1 with some proper scaling. A node can receive a message only if exactly one of its neighbors sends.

In the collision detection model, nodes are able to sense the existence of transmissions while listening. In particular, for the UDG model, a node v capable of collision detection means that when v listens in round i, if it has at least one transmitter in its neighborhood $N(v)$, it can detect a transmission.

We also give the definition of the maximal independent set which will be used in the algorithm. In a graph $G = (V, E)$, a set S is a maximal independent set if any two nodes $u, v \in S$ have hop distance of at least 2 and every node $v \in V \setminus S$ is adjacent to a node $u \in S$.

B. Synchronous Wake-up

Our algorithm follows the standard broadcast strategy of cycling through selection objects. A selection object is a schedule of every node which guarantees that each node performing broadcasting will be isolated. To do this, we compute

a maximal independent set of the network by modeling it as a "unit" disc graph using a carefully chosen scaling factor R.

When adopting the uniform power assignment, the network can be clearly modeled as a unit disc graph $G = (V, E, R_T)$, where there exists an edge $(u, v) \in E$ between two nodes u and v if and only if $d(u, v) \leq R_T$. However, this is still different from the theoretical UDG model, since the cumulative interference may make a node v fail to receive a message even when only one neighbor of v sends. Due to this difference, it is difficult to simply adopt some distributed MIS algorithms designed for the UDG model, in which a node makes its decision based on its previous states and received messages in the current round. Instead, we adopt a collision detection based algorithm like the one in [36].

In the collision detection UDG model, the deterministic MIS algorithm in [36] can compute a MIS in $O(\log n)$ synchronized rounds under the condition that all nodes have unique IDs from the interval $[1, n]$, where n is an estimate of the number of nodes. Another advantage of the algorithm is that the whole algorithm is just based on the utilization of collision detection, and does not depend on the successful delivery of messages in any way. Hence, when assuming that nodes can perform physical carrier sensing, we can implement the collision detection based MIS algorithm to compute a MIS in the following defined sensitivity graph, which is clearly a UDG under the scaling factor R_S determined by P.

Definition 2 (Sensitivity Graph): For transmission power level P, the sensitivity graph $G_S = (V, E_S)$ is defined on the node set V. An edge uv exists in G_S for two nodes u, v if and only if $d(u, v) \leq R_S$, where R_S is the carrier sensing range determined by P according to (2).

Next we introduce the deterministic local broadcasting algorithm demonstrated in Algorithm 2. The algorithm is performed in synchronized rounds between all nodes. It is divided into stages (corresponding to each loop). Each stage consists of $t_{mis} + 1$ rounds, where t_{mis} is the time Algorithm MIS in [36] takes for computing a MIS. In each stage, Algorithm MIS is adopted to compute a MIS for the sensitivity graph defined by a carefully chosen transmission power level P_1. After finishing the implementation of Algorithm MIS, every node either joins state $\mathcal{M}$ which means that it is one member of the computed MIS, or otherwise joins state $\mathcal{N}$. All nodes in state $\mathcal{M}$ will perform a local broadcast with transmission power P_2 in the subsequent one round and terminate performing the algorithm after that. Then a new stage starts and all nodes with state $\mathcal{N}$ will execute the algorithm again. At first, all nodes are in state $\mathcal{N}$.

Let $P_2 = 2N\beta R_B^\alpha$ and $P_1 = TR_S^\alpha$, where $R_S \geq [96\beta(2^{\alpha-1} + \frac{\alpha-1}{\alpha-2})]^{\frac{1}{\alpha}} R_B$ and T is the hardware-defined carrier sensing threshold. Here we choose R_S such that $R_S \geq 2R_A$.

Lemma 7 ([36]): The total time to compute a MIS in each stage is $O(\log n)$.

Using a similar idea as that for proving Lemma 3, we can show the correctness of the following lemma by bounding the interference layer by layer. The detailed proof can be found in the full version [37].

Algorithm 2 Synchronous Waking-up Local Broadcasting With Carrier Sensing

For each node $v \in V$
1: **loop**
2:   **if** $state_v := \mathcal{N}$
3:   Perform MIS algorithm [36] with power P_1.
4:     **if** $state_v := \mathcal{M}$
5:      $transmit()$ with power P_2;
6:     **else** wait for a round and $state_v := \mathcal{N}$.
7:     **end if**
8:   **end if**
9: **end loop**

Lemma 8: Each node v in state $\mathcal{M}$ can successfully perform a local broadcast under the SINR constraint.

Theorem 2: Denote by B_v^S the disc centered at v with radius R_S and by Δ_v^S the number of nodes in B_v^S. After $O(\Delta_v^S \log n)$ rounds, each node v will successfully perform a local broadcast. Furthermore, Algorithm 2 can achieve an approximation ratio $O(\log n)$ in time complexity.

Proof: By Lemma 8, at the end of each stage, all nodes in state $\mathcal{M}$ will carry out a local broadcast successfully in one round. By Lemma 7, we know that the MIS algorithm can correctly compute a MIS in $O(\log n)$ rounds, which means that in each stage, for each node v, at least one node in B_v^S will carry out a local broadcast successfully. Thus, after at most Δ_v^S stages, v will join state $\mathcal{M}$ and perform a local broadcast in B_v. The total time is at most $\Delta_v^S \cdot O(\log n + 1) = O(\Delta_v^S \log n)$. The approximation ratio of Algorithm 2 can be obtained in a similar manner to that in Theorem 1. ∎

C. Asynchronous Wake-up

As discussed in [36], asynchronism introduces some difficulties. A typical one is that when a node wakes up and transmits without knowing the state of its neighbors which may disturb and corrupt an ongoing computation of a MIS. For the asynchronous wakeup scenario, the authors extended the MIS algorithm for the synchronously waking-up version by performing a six-round scheduling repeatedly. Here we generalize their Asynchronous MIS Algorithm to achieve local broadcasting for every node in the asynchronously waking-up scenario.

The basic idea of the Asynchronous MIS Algorithm is that nodes involved in a computation (or in a MIS) transmit periodically to force newly waking-up nodes to wait. Upon waking up, a node listens until no neighbors have transmitted for 7 consecutive rounds. If a node has detected transmission for two consecutive rounds it knows that there is a neighbor in the MIS. Then it changes its state to $\mathcal{N}$ and will not compete to join the MIS. If it has not detected any transmission for 7 consecutive rounds, the node will execute Algorithm MIS [36] by iterating a six-round scheduling. In the six-round scheduling, a node which is executing or is about to execute Algorithm MIS transmits in the first round. This ensures that each of its neighbors either starts executing Algorithm MIS

concurrently or waits until it has completed the algorithm. Then the node will execute one step in Algorithm MIS. In the second and fourth rounds no transmissions occurs. If a node is in the MIS, it transmits in the fifth and sixth rounds. The schedule is repeated endlessly such that nodes in the MIS continuously inform waken-up neighbors about their presence. The following Lemma is proved in [36].

Lemma 9: The complexity of the Asynchronous MIS Algorithm is $O(\log n)$.

Algorithm 3 Asynchronous Waking-up Local Broadcasting With Carrier Sensing

For each node $v \in V$, **upon wake-up**
1: $state_v := \mathcal{W}$;

node v in state $\mathcal{W}$
1: **Listen until** no transmission sensed for 8 consecutive rounds **then** $state_v := \mathcal{A}$

node v in state $\mathcal{A}$
Loop
1: **if** $state_v := \mathcal{A}$ **then** Transmit with power P_1 **else** sleep **end if**
2: sleep
3: **if** $state_v := \mathcal{A}$ **then** Execute one step in Algorithm MIS in [36] with transmission power P_1 **else** sleep **end if**
4: sleep
5: **if** $state_v := \mathcal{M}$ **then** Transmit twice with power P_1
6: **else** carry out carrier sensing for two rounds
7:   **if** v detected transmission for 2 consecutive rounds **then** $state_v = \mathcal{W}$ **end if**
8: **end if**
9: **if** $state_v := \mathcal{M}$ **then** Transmit with power P_2 and quit **else** listen **end if**
end loop

In Algorithm 3, the values of P_1 and P_2 are the same as those assigned in Section IV-B. Upon waking up, a node v enters the waiting state $\mathcal{W}$ and listens until no neighbors have transmitted for 8 consecutive rounds. Then it joins state $\mathcal{A}$ to compete for entering an independent set of the sensitivity graph defined by P_1, i.e., joining state $\mathcal{M}$. After entering state $\mathcal{A}$, node v will repeat a seven-round scheduling. Different from the six-round scheduling in the Asynchronous MIS Algorithm, in the five and six rounds, all nodes that sensed a neighboring node in state $\mathcal{M}$ will join state $\mathcal{W}$ and restart the algorithm. Furthermore, if v joins state $\mathcal{M}$, it will stop executing the algorithm after performing a local broadcast with power P_2 in the seventh round such that its neighbors can restart the algorithm to compete for joining an independent set to complete a local broadcast. The Asynchronous MIS Algorithm in [36] guarantees that in any round, the nodes in state $\mathcal{M}$ constitutes an independent set, since nodes restarting executing the algorithm can be seen as newly waken-up nodes. Lemma 9 tells us that for each node v, after at most $O(\log n)$ rounds from the time it starts or restarts executing the algorithm, there will emerge a node in its carrier sensing region joining

state $\mathcal{M}$. Furthermore, by Lemma 8, all nodes in state $\mathcal{M}$ can perform local broadcasting successfully. Finally, using a similar analysis as that in Theorem 2, the following result can be obtained.

Theorem 3: Upon waking up, each node v will successfully perform a local broadcast after $O(\Delta_v^S \log n)$ rounds. Furthermore, Algorithm 3 achieves an approximation ratio $O(\log n)$ in time complexity.

V. Conclusions

In this paper, by using a more realistic physical interference model and by considering both the synchronized and asynchronized communication models, we have presented three distributed local broadcasting algorithms. For the asynchronized model, our proposed distributed randomized algorithm outperforms the state-of-the-art result [14] by a logarithmic factor. For the synchronized model, considering both synchronous and asynchronous node wakeups, our two proposed algorithms are the first distributed deterministic algorithms for the local broadcasting problem under the SINR model. Note that we have assumed that each sensor node employs the same transmission power (i.e., uniform power assignment). The question then is whether we can design efficient distributed algorithms for the local broadcasting problem with the presence of power control, i.e., different nodes may use different powers to transmit. This should be a meaningful challenge to take up as power control can significantly reduce the time complexity [9], [32]. Another possible direction is to design distributed deterministic algorithms for the local broadcasting problem under the asynchronized communication model.

VI. Acknowledgement

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